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AUTHOR: Vásquez Terreros, Jorge Luis

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SURNAMES: Vásquez Terreros **NAMES: Jorge Luis**

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DIRECTOR: Pau Martí Colom

DEPARTAMENT: Department of Systems, Automation and Industrial Informatics Engineering

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ABSTRACT

Microgrids (MGs) are cataloged as Low-Voltage (LV) distribution networks which comprise distributed generators (DGs), energy storage devices and controllable loads that can operate in grid connected and islanded mode. The objective of this thesis is to introduce an alternative MG modeling technique, different from known modeling methods, which may allow us to obtain new control algorithms applicable to MGs. This modeling technique based on complex power will be validated according to the results obtained with the simulations of the model based on the Modified Nodal Analysis (MNA) method that is based on currents and voltages.
To show this modeling technique, we will consider an MG composed by two three-phase inverters feeding a resistive load (MG-2Inv-3f-RL) in islanded mode.
We will demonstrate that the modeling technique is correct and feasible by implementing the MNA-based method and the complex-based method. We will check the complexity that the MNA-based modeling method acquires.
To achieve our goal, we will review the concepts of the MNA-based modeling method applied to a basic electrical circuit. We will use the Symbolic Circuit Analysis in Matlab (SCAM) tool based on MNA, to automate the modeling technique of a MG. We will perform the open-loop analysis and simulation of the MNA-based MG model and the complex-based MG model.
Finally, after analyzing the stability of the complex-based model, we will proceed to perform the closed-loop analysis and simulation implementing the droop control strategy in the MNA-based model and the complex-based model. Hence, we will verify the validity of the alternative complex-based MG modeling technique.

Keywords:

Low-Voltage (LV)	Distributed Generators (DGs)
Microgrids (MGs)	Modified Nodal Analysis (MNA)
Symbolic Circuit Analysis in Matlab (SCAM)	Control
Droop	Model

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1 Introduction

The continuous technological advances and the establishment of environmental policies worldwide have motivated, since some years ago, research on alternative energy generation sources such as renewables that reduces global emissions of carbon and those derived from fossil fuels into the environment by the units typical power generation plants. In addition, the increase in equipment, devices, computer applications and industrial processes in all areas demand a greater amount of energy in the large interconnected networks which could result in failures and supply cuts. These factors have prompted researchers to delve into modern power systems, since places where there is an over-demand for electricity until places where there is still no access to it. In this context, Microgrids (MGs) have the potential to support many problems faced by today's power systems. Microgrids (MGs) are defined as low-voltage (LV) distribution networks which comprise of various distributed generation (DGs) units (photovoltaic arrays (PV), wind generators, fuel-cells, microturbines, etc.), energy storage devices (flywheels, super-capacitors and batteries) and controllable loads that can operate interconnected or isolated from the large electrical network, and that can be collectively treated by the network as a controllable load or generator [1]. The role of the energy storage devices in MG is to strike a balance between energy generation and consumption especially during sudden changes in load or generation [2].

In addition, most renewable DG units are interfaced to the network via DC/AC inverters. The physical characteristics of such power electronic devices largely differ from the characteristics of synchronous generators (SGs), which are the standard generating units in existing power systems. Hence, different control and operation strategies are needed in networks with a large amount of renewable DG units [3].

A MG can be operated in two modes, such as grid-connected (on-grid) mode and islanded (off-grid, emergency, autonomous) mode. In grid-connected mode, the MG is connected to the main power grid and either receives power or injects some power into it. In islanded mode, the MG is disconnected from the main grid and it operates autonomously like physical islands [2].

Many new control problems arise for this type of networks and their satisfactory solution may require the development of advanced controller design techniques that will benefit from the availability of rigorous MG models. A recent review on MG modeling can be found in [3].

Controlling in MG, mainly involves either controlling the active and reactive power flowing through power inverters. Among the control strategies we have the PQ control scheme (grid-feeding mode) that adjusts the output voltage amplitude and phase to control the active and reactive power.A review on MG control research can be found in [2].

The purpose of this work is to introduce an alternative modeling technique for complex-based MGs, different from known modeling methods, which may allow us to generate new control algorithms to apply them to MGs. The results obtained with this complex-based modeling technique will be validated with the results acquired by the Modified Nodal Analysis (MNA) method using the same MG parameters.

The MNA method uses Kirchoff's current law and is algorithmically more efficient than the mesh current method or the node voltage method [5]. We will review the concepts of the MNA-based modeling method and apply them to a basic electrical circuit and to a MG composed by two single-phase inverters feeding a resistive load.

We will automate the MNA-based MG modeling technique with the Symbolic Circuit Analysis in MatLab (SCAM) tool. This Matlab script allows us to symbolically solve systems of equations in electrical circuits using a netlist file that defines the interaction of the components in the MG nodes [6].

The result that we obtain with the MNA-based modeling method is determined by the expression $I(s) = G(s)V(s)$ where $I(s)$ are the system outputs corresponding to the currents of each inverter, $G(s)$ is the system transfer function and $V(s)$ are the system inputs corresponding to the voltages of each inverter.

To resulting expression cited above, we structure it in a transfer function model and later, we express it in a state space model. In this way, we can obtain the power of each inverter as the product of the current and the voltage and we simplify other calculation processes.

The analysis and simulation results in open loop that we obtain with the MNA-based modeling method and the alternative complex-based modeling technique of a MG composed by two three-phase inverters feeding a resistive load, allow us to preliminarily validate the latter.

Finally, we discuss a stability analysis of the complex-based MG model. We implement the droop control strategy to control and balance the power that each MG inverter generates in the MNA-based model and in the complex-based model [16].
The closed-loop simulation result of the model obtained with the complex-based modeling technique and the MNA-based modeling method show the

effect of the droop method and the equal power sharing function. These results also indicate the feasibility and confirm the validity of the alternative complex-based MG modeling technique that we propose in this work.

The figure 1 illustrates the structure of the topics to be treated in this work.

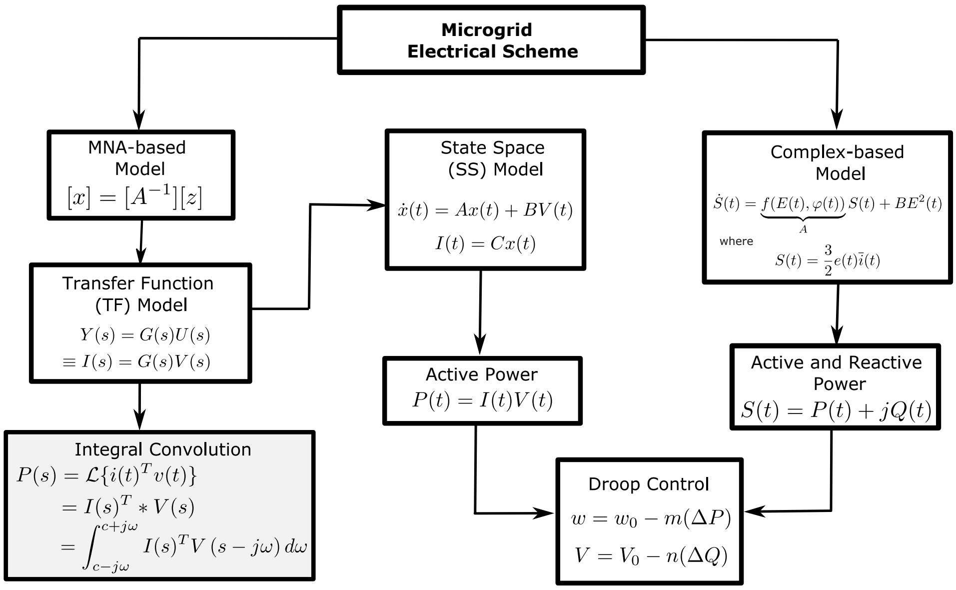


Figure 1: Main scheme

1.1 Objectives

1.1.1 General Objective

Introduce an alternative MG modeling technique, different from known modeling methods, which allow us to obtain new control algorithms applicable to MGs.

1.1.2 Specific Objectives

- Review the Modified Nodal Analysis Method (MNA) with a basic electrical circuit, applying a manual analysis.
- Review the Symbolic Circuit Analysis in Matlab (SCAM) tool with a basic electrical circuit, applying the MNA algorithms in the Matlab's script, and compare with the manual analysis.
- Apply the MNA method to a MG of two single-phase inverters with a resistive load, performing a manual analysis and an automated analysis using SCAM in Matlab, and compare the results.

- Convert the result obtained with the MNA-based MG modeling method in Transfer Function (TF) model, to later transform it to State Space (SS) model.
- Simulate and analyze the open loop behavior of the MG model of two single-phase inverters with a resistive load, when the inverters' voltage and phase parameters are varied during runtime.
- Analyze and simulate the open-loop behavior of a MNA-based MG model of two three-phase inverters with an resistive load, using SCAM and Matlab functions, when the inverters' voltage and phase parameters are varied during runtime.
- Propose, analyze and simulate the open-loop behavior of a Complex-based MG model of two three-phase inverters with an resistive load using Matlab, and compare the results of the two models.
- Preliminary stability analysis of complex-based MG model.
- Implement and simulate the droop control strategy in the complex-based and MNA-based MG model using Matlab, and compare the results of the two models.

1.2 Document Structure

The document is organized as follows. Section 2 describes the Modified Nodal Analisys (MNA) method used to obtain and solve the differential equations that represent the interconnection of the components of a basic electrical circuit. The concept of the MNA method is explained with a basic electrical circuit and with MG model of two single-phase inverters with a resistive load, modeling the MGs in state space. Section 3 shows the MNA-based modeling method and simulation of a MG of two three-phase inverters with a resistive load (MG-2Inv-3f-RL). Discuss the respective results. Section 4 shows the complex-based modeling technique and simulation of MG-2Inv-3f-RL in the $\alpha\beta$ frame, with the respective results and their valuation. Section 5 describes the stability analysis of complex-based MG model and shows the effect of droop control strategy applied to MNA-based and complex-based MG model in closed-loop. Discuss the validity of complex-based MG modeling technique. Finally, Section 6 presents the conclusions of this work.

2 Modified Nodal Analysis (MNA)

2.1 MNA Description

Despite the fact that the node voltage method and the mesh current method are widely used in solving systems of equations related to electrical circuits, there is a very practical and powerful method known as Modified Nodal Analysis (MNA). The systems of equations obtained as a result of the MNA, in general, are larger than the other methods mentioned, but it can be implemented algorithmically more easily on a computer, being a fundamental advantage to obtain automated solutions. In general terms, to apply the MNA, an equation is written for each of the nodes not connected to a voltage source and these equations are increased with an equation for each voltage source [5].

The steps that are considered to apply the modified nodal analysis to a circuit with n nodes (with m voltage sources) are presented below:

1. Identify the number of nodes (n) in the circuit, select a reference node (usually ground) and mark the remaining nodes ($n-1$). Also indicate the currents through each current source if you have them.

2. Label the current through each voltage source. We are going to use the current flow convention from the positive node to the negative node of the voltage source.

3. Apply Kirchhoff's current law to each node. We will mark the currents out of the node to be considered positive.

4. Write an equation for the voltage of each voltage source.

5. Solve the system of $(n-1+m)$ unknowns.

2.1.1 Manual analysis applied to a simple electrical circuit

To put into practice the steps detailed previously, consider the simple resistive circuit with independent voltage of figure 2 as an example:

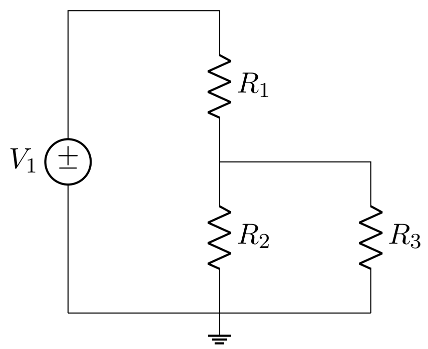


Figure 2: Example of an electrical circuit proposed for the application of MNA

We apply, in figure 3, step 1 identifying the nodes of the electrical circuit and assigning them a label where ground is reference node and the other nodes named consecutively, each node being arbitrarily chosen.

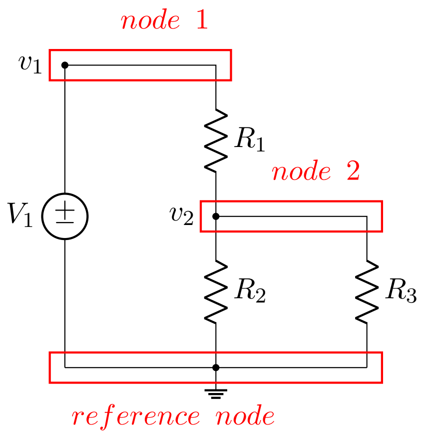


Figure 3: Definition of nodes in the electric circuit proposed for the application of MNA

The step 2 is applied, defining the current that passes through the voltage sources in the direction of the positive node to the negative node, as shown in electrical circuit of figure 4.

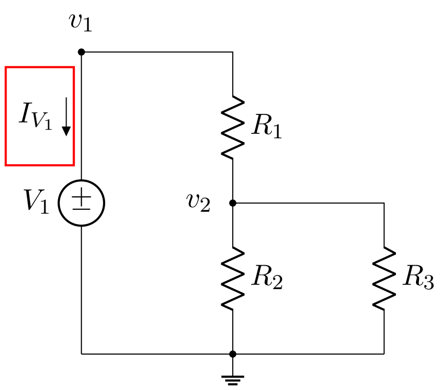


Figure 4: Definition of currents through voltage sources in the electrical circuit proposed for the application of MNA

Proceed to step 3, applying Kirchoff's current law (**KCL**) to each node and considering the outgoing currents of the node as positive, as shown in the figure 5 and it is observed in more detail in the table 1.

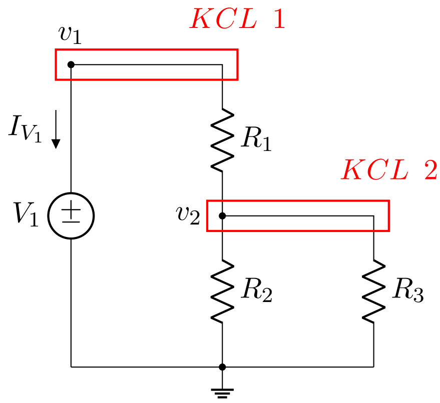


Figure 5: Application of kirchhoff's current law to each node in the electrical circuit proposed for the application of MNA

After concluding with step 3, a voltage equation is determined in step 4 for each voltage source and, as the fifth and last step, we proceed to solve the system of equations obtained from the electrical circuit, which is illustrated in the table 1.

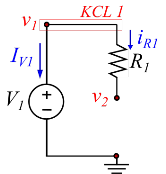
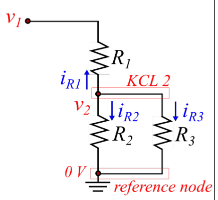
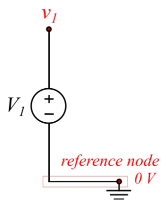
Electrical circuit section	Node equation	Matrix equation $\mathbf{Ax} = \mathbf{z}$
	$I_{V1} + i_{R1} = 0$ $\Rightarrow I_{V1} + \frac{v_1 - v_2}{R_1} = 0$ $\Rightarrow \frac{v_1}{R_1} - \frac{v_2}{R_1} + I_{V1} = 0$	$\mathbf{A} = \begin{bmatrix} \mathbf{\frac{1}{R_1}} & -\mathbf{\frac{1}{R_1}} & \mathbf{1} \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \end{bmatrix}$ $\mathbf{x} = \begin{bmatrix} v_1 \\ v_2 \\ I_{V1} \end{bmatrix}$ $\mathbf{z} = \begin{bmatrix} \mathbf{0} \\ \vdots \\ \vdots \end{bmatrix}$
	$i_{R1} + i_{R2} + i_{R3} = 0$ $\Rightarrow \frac{v_2 - v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_2}{R_3} = 0$ $\Rightarrow -\frac{v_1}{R_1} + \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) v_2 = 0$	$\mathbf{A} = \begin{bmatrix} \frac{1}{R_1} & -\frac{1}{R_1} & 1 \\ -\mathbf{\frac{1}{R_1}} & \left(\mathbf{\frac{1}{R_1}} + \mathbf{\frac{1}{R_2}} + \mathbf{\frac{1}{R_3}}\right) & \mathbf{0} \\ \vdots & \vdots & \vdots \end{bmatrix}$ $\mathbf{x} = \begin{bmatrix} v_1 \\ v_2 \\ I_{V1} \end{bmatrix}$ $\mathbf{z} = \begin{bmatrix} 0 \\ \mathbf{0} \\ \vdots \end{bmatrix}$
	$v_1 - 0 = V_1$ $\Rightarrow v_1 = V_1$	$\mathbf{A} = \begin{bmatrix} \frac{1}{R_1} & -\frac{1}{R_1} & 1 \\ -\frac{1}{R_1} & \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) & 0 \\ \mathbf{1} & \mathbf{0} & \mathbf{0} \end{bmatrix}$ $\mathbf{x} = \begin{bmatrix} v_1 \\ v_2 \\ I_{V1} \end{bmatrix}$ $\mathbf{z} = \begin{bmatrix} 0 \\ 0 \\ \mathbf{V_1} \end{bmatrix}$

Table 1: Process of obtaining the system of equations of the electrical circuit

MNA Method applied to a circuit with elements such as*resistors, inductors, capacitors and/or operational amplifiers* ***op-amps*** and independent current and voltage sources results in a matrix equation of the form:

$$\mathbf{Ax} = \mathbf{z} \tag{2.1}$$

The formation of matrix equation (2.1) is illustrated step by step in detail in the third column of the table 1.

$$\underbrace{\begin{bmatrix} \frac{1}{R_1} & -\frac{1}{R_1} & 1 \\ -\frac{1}{R_1} & \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) & 0 \\ 1 & 0 & 0 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} v_1 \\ v_2 \\ I_{V1} \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ V_1 \end{bmatrix}}_{\mathbf{z}} \tag{2.2}$$

The expression's matrices obtained in (2.2) will also be obtained later in the section 2.1.3, through computational algorithms using Matlab, as shown in (2.28), (2.29) and (2.30).

To solve the equation of the circuit, a manipulation of the matrix equation (2.1) is carried out, obtaining the equation (2.3):

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{z} \tag{2.3}$$

The difficulty in solving this type of equation systems increases when a manual analysis is performed according to the complexity of the electrical circuit, but it is very easy and fast if we make use of computational tools such as Matlab, as follows from (2.3):

$$\underbrace{\begin{bmatrix} v_1 \\ v_2 \\ I_{\mathrm{V1}} \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} 0 & 0 & 1 \\ 0 & \frac{R_1\,R_2\,R_3}{R_1\,R_2+R_1\,R_3+R_2\,R_3} & \frac{R_2\,R_3}{R_1\,R_2+R_1\,R_3+R_2\,R_3} \\ 1 & \frac{R_2\,R_3}{R_1\,R_2+R_1\,R_3+R_2\,R_3} & -\frac{R_2+R_3}{R_1\,R_2+R_1\,R_3+R_2\,R_3} \end{bmatrix}}_{\mathbf{A^{-1}}} \underbrace{\begin{bmatrix} 0 \\ 0 \\ V_1 \end{bmatrix}}_{\mathbf{z}} \tag{2.4}$$

From (2.4), we note that we have obtained the inverse of matrix **A** and solving the matrix equation for **x** symbolically using Matlab's commands `x = inv(A) * z`, we have:

$$\mathbf{x} = \begin{bmatrix} v_1 \\ v_2 \\ I_{\mathrm{V1}} \end{bmatrix} = \begin{bmatrix} V_1 \\ \frac{R_2\,R_3\,V_1}{R_1\,R_2+R_1\,R_3+R_2\,R_3} \\ -\frac{V_1\,(R_2+R_3)}{R_1\,R_2+R_1\,R_3+R_2\,R_3} \end{bmatrix} \tag{2.5}$$

The response of equations system, reflected in expression (2.5), will also be obtained through computational algorithms in Matlab as shown in (2.31), in section 2.1.3.

2.1.2 Matricial algorithmic MNA applied to a simple electrical circuit

There is a way to use the MNA method algorithmically through the matrix representation of graph theory known as the Laplacian matrix, admittance matrix or Kirchhoff matrix [10] which helps us determine many properties of graphs or circuits together with Kirchhoff's theorem that we compete. In this particular case, applied to electrical circuits with independent sources of voltage and current, **A** matrix from expression (2.1) is formed as a combination of four smaller matrices or sub-matrices **G**, **B**, **C**, **D** [11] and it is $(n+m)\times(n+m)$ where **n** is the **number of nodes** without considerer reference node, and **m** is the **number of independent voltage sources** (2.6).

$$\mathbf{A} = \begin{bmatrix} \mathbf{G} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \tag{2.6}$$

- The **G** sub-matrix ($n \times n$) is determined by the interconnections between the passive circuit elements (resistors, inductors, etc).
  - Each element in the main diagonal of matrix **G** is equal to the sum of the admittances (one over the impedance) of each element connected in the respective node, that is, the first element of the main diagonal corresponds to the sum of the admittances

connected to node 1, the second element of the main diagonal corresponds to the sum of the admittances connected to node 2, and so on.

- The elements off the diagonal correspond to the negative admittance of the element connected to the respective pair of nodes, that is, the negative admittance of the element connected between node 1 and node 2 would go in the position $(1, 2)$ and $(2, 1)$ of matrix $\mathbf{G}$, and so on.

According to the example electrical circuit in figure 3, the matrix $\mathbf{G}$ that we obtain is shown at (2.7).

$$\mathbf{G} = \begin{bmatrix} \frac{1}{R_1} & -\frac{1}{R_1} \\ -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \end{bmatrix} \tag{2.7}$$

- The $\mathbf{B}$ sub-matrix $(n \times m)$ is determined by the connection of the voltage sources.
  - The elements of $\mathbf{B}$ sub-matrix only consist of 0, 1 and -1. Each column represents each voltage source respectively, while each row represents each node to which the terminals of the sources are connected. If the positive terminal is connected to a node, a 1 is placed at that element's position in the matrix. If the negative terminal of the source is connected, -1 is placed in that corresponding element. Any other case, the elements of the matrix are 0.

Returning to our example, the $\mathbf{B}$ sub-matrix obtained by applying this rule is shown in (2.8).

$$\mathbf{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{2.8}$$

- The $\mathbf{C}$ sub-matrix $(m \times n)$ is determined by the connection of the voltage sources. When independent sources are considered, then $\mathbf{B}$ and $\mathbf{C}$ interrelate, in fact, $\mathbf{C}$ becomes the transposed $\mathbf{B}$ sub-matrix $(\mathbf{C} = \mathbf{B}^T)$.
  - The rule mentioned in the previous paragraph does not apply when dependent sources are present in the circuit.

In our example, where we have an independent voltage source, the $\mathbf{C}$ sub-matrix is presented at (2.9).

$$\mathbf{C} = \mathbf{B}^T = \begin{bmatrix} 1 & 0 \end{bmatrix} \tag{2.9}$$

- The **D** sub-matrix ($m \times m$) is **0** if only independent sources are considered.
  - The rule mentioned in the previous paragraph may not apply when there are dependent sources in the circuit.

  In our case, the **D** sub-matrix is composed of 0, as shown in (2.10).

$$\mathbf{D} = \begin{bmatrix} 0 \end{bmatrix} \tag{2.10}$$

From the sub-matrices obtained **G**, **B**, **C** and **D**, we generate the block **A** matrix in (2.11) being able to observe that we obtain the same **A** matrix shown in (2.2), but in an algorithmic way through the graph.

$$\mathbf{A} = \begin{bmatrix} \frac{1}{R_1} & -\frac{1}{R_1} & 1 \\ -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & 0 \\ 1 & 0 & 0 \end{bmatrix} \tag{2.11}$$

The **x** matrix contains the unknowns of equations system of the circuit and is formed by two smaller matrices or sub-matrices **v** and **j**, and this is of order $((m+n) \times 1)$ where **n** is the **number of nodes** without considerer reference node, and **m** is the **number of independent voltage sources** (2.12).

$$\mathbf{x} = \begin{bmatrix} \mathbf{v} \\ \mathbf{j} \end{bmatrix} \tag{2.12}$$

- The **v** sub-matrix ($n \times 1$) contains the unknowns of the voltages of each respective node. In the proposed example of two nodes we obtain the expression in (2.13).

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{2.13}$$

- The **j** sub-matrix ($m \times 1$) contains the unknown of the current that passes through each voltage source, therefore, from the example with a voltage source, we have the expression in (2.14).

$$\mathbf{j} = \begin{bmatrix} I_{\text{V1}} \end{bmatrix} \tag{2.14}$$

From the sub-matrices obtained **v** and **j**, we generate the block **x** matrix in (2.15) being able to observe that we obtain the same **x** matrix shown in (2.2)

$$\mathbf{x} = \begin{bmatrix} v_1 \\ v_2 \\ I_{\text{V1}} \end{bmatrix} \tag{2.15}$$

The $\mathbf{z}$ matrix contains the independent voltage and current sources and is formed by two smaller matrices or sub-matrices $\mathbf{i}$ and $\mathbf{e}$, and this is of order $((m+n) \times 1)$ where $\mathbf{n}$ is the **number of nodes** without considerer reference node, and $\mathbf{m}$ is the **number of independent voltage sources** (2.16).

$$\mathbf{z} = \begin{bmatrix} \mathbf{i} \\ \mathbf{e} \end{bmatrix} \tag{2.16}$$

- The $\mathbf{i}$ sub-matrix $(n \times 1)$ contains the sum of the value of the independent current sources in the corresponding node or 0 in the absence of any. In the proposed example of two nodes we obtain the expression in (2.17).

$$\mathbf{i} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{2.17}$$

- The $\mathbf{e}$ sub-matrix $(m \times 1)$ contains each element of the sub-matrix equal to the value of the respective independent voltage source, therefore, from the example with a voltage source, we have the expression in (2.18).

$$\mathbf{e} = \begin{bmatrix} V1 \end{bmatrix} \tag{2.18}$$

From the sub-matrices obtained $\mathbf{i}$ and $\mathbf{e}$, we generate the block $\mathbf{z}$ matrix in (2.19) being able to observe that we obtain the same $\mathbf{z}$ matrix shown in (2.2)

$$\mathbf{z} = \begin{bmatrix} 0 \\ 0 \\ V_1 \end{bmatrix} \tag{2.19}$$

Finally, to obtain the solution of the equations system of an electric circuit algorithmically, we use the method of block matrix inversion [11] in matrix A (2.6) such that, if it is partitioned into 4 blocks as is our case, it can be inverted blockwise (2.20) as follows:

$$\mathbf{A^{-1}} = \begin{bmatrix} \mathbf{G} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{G^{-1}} + \mathbf{G^{-1}B}\left(\mathbf{D} - \mathbf{CG^{-1}B}\right)^{-1}\mathbf{CG^{-1}} & -\mathbf{G^{-1}B}\left(\mathbf{D} - \mathbf{CG^{-1}B}\right)^{-1} \\ -\left(\mathbf{D} - \mathbf{CG^{-1}B}\right)^{-1}\mathbf{CG^{-1}} & \left(\mathbf{D} - \mathbf{CG^{-1}B}\right)^{-1} \end{bmatrix} \tag{2.20}$$

where $\mathbf{G}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ can be arbitrary size but $\mathbf{G}$ and $\mathbf{D}$ must be square matrices so that they can be inverted. Furthermore, $\mathbf{G}$ and $(\mathbf{D} - \mathbf{CG^{-1}B})$ must be invertible.

Now, we are going to obtain a direct expression to obtain the currents of each independent voltage source in the electrical circuit. What is done first is to multiply both members of the expression in (2.3) by a matrix in such a way that the voltage variables that I do not want to find are canceled and I leave the current variable in (2.15) that we require by means of the identity matrix I depending on the number of voltje sources (2.21) (2.22).

$$\mathbf{Sx} = \mathbf{SA}^{-1}\mathbf{z} \tag{2.21}$$

where:

$$\begin{aligned} \mathbf{S} &= \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix} \\ \Rightarrow \mathbf{S} &= \begin{bmatrix} \underbrace{0}_{n\ nodos} & \underbrace{I}_{m\ fuentes} \end{bmatrix} \end{aligned} \tag{2.22}$$

Performing the operation on the term $\mathbf{Sx}$ we have (2.23)

$$\mathbf{Sx} = \begin{bmatrix} I_{V_m} \end{bmatrix} \tag{2.23}$$

where the elements of current matrix depends on the $\mathbf{m}$ number of independent voltage sources in the circuit. In the same way, with the term $\mathbf{SA^{-1}z}$ we have (2.24).

$$\begin{aligned} \mathbf{SA}^{-1}\mathbf{z} &= \begin{bmatrix} 0 & I \end{bmatrix} \begin{bmatrix} \mathbf{G^{-1}} + \mathbf{G^{-1}B}\left(\mathbf{D} - \mathbf{CG^{-1}B}\right)^{-1}\mathbf{CG^{-1}} & -\mathbf{G^{-1}B}\left(\mathbf{D} - \mathbf{CG^{-1}B}\right)^{-1} \\ -\left(\mathbf{D} - \mathbf{CG^{-1}B}\right)^{-1}\mathbf{CG^{-1}} & \left(\mathbf{D} - \mathbf{CG^{-1}B}\right)^{-1} \end{bmatrix} \begin{bmatrix} 0 \\ V_m \end{bmatrix} \\ &= \begin{bmatrix} -\left(\mathbf{D} - \mathbf{CG^{-1}B}\right)^{-1}\mathbf{CG^{-1}} & \left(\mathbf{D} - \mathbf{CG^{-1}B}\right)^{-1} \end{bmatrix} \begin{bmatrix} 0 \\ V_m \end{bmatrix} \\ &= \begin{bmatrix} \left(\mathbf{D} - \mathbf{CG^{-1}B}\right)^{-1} \end{bmatrix} \begin{bmatrix} V_m \end{bmatrix} \end{aligned} \tag{2.24}$$

where the elements of voltage matrix depends on its $\mathbf{m}$ number of independent voltage sources in the circuit as in (2.23). Then the expression to find the current using this method is represented in (2.25).

$$\begin{bmatrix} I_{V_m} \end{bmatrix} = \begin{bmatrix} \left(\mathbf{D} - \mathbf{CG^{-1}B}\right)^{-1} \end{bmatrix} \begin{bmatrix} V_m \end{bmatrix} \tag{2.25}$$

After obtaining an expression in (2.25) that allows us to calculate the currents that pass through independent voltage sources in an electric circuit, we verify in (2.26) that the algorithmic process using the Laplacian matrix and the inversion of the block matrix in $\mathbf{A}$ matrix gives us the same current values obtained in (2.5) of section 2.1.1 and in (2.31) of section 2.1.3 with this example using Matlab as follows:

```
syms R1 R2 R3 V1
G=[1/R1                 -1/R1 ;
    -1/R1       1/R1+1/R2+1/R3];
B=[1;
   0];
C=[1 0]; %C=B';
D=[0];
I_V1=[(D-C*G^-1*B)^-1]*V1
```

```
I_V1 =
-(V1*(R2 + R3))/(R1*R2 + R1*R3 + R2*R3)

       V1 (R2 + R3)
- ---------------------
  R1 R2 + R1 R3 + R2 R3
```

Algorithm 1: Matrix algorithm solution of current vs voltage in Matlab's editor and command window

$$I_{V_1} = -\frac{V_1\ (R_2 + R_3)}{R_1\, R_2 + R_1\, R_3 + R_2\, R_3} \tag{2.26}$$

In this way we can directly obtain the currents as a function of the voltages in the transfer function model with the expression $Y(s) = G(s)U(s)$, where $U(s)$ are the system inputs corresponding to the inverter voltages, $G(s)$ is the transfer function of the system and $Y(s)$ are the outputs corresponding to the currents that pass through each source or inverter.

2.1.3 SCAM computer analysis applied to a simple electrical circuit

As it is required to model and analyze MGs from multiple inverters, then the scalability and ease of being able to deal algorithmically with systems of very large equations is essential, and for this reason the use of a computational tool is required to automate the process of solving equations such as Matlab, which is a very powerful software to carry out all this type of analysis and simulations at an academic and professional level. SCAM (Symbolic Circuit Analysis in MatLab) describe the MATLAB tool that performs MNA (Modified Nodal Analysis) for deriving and solving equations systems symbolically on a electrical circuit, given a netlist file that defines the interconnection between circuit components.

The code is written as a MATLAB script instead of a function because it would make learning easier because it can be stepped through, and all of the variables created by the code appear in the workspace so users can examine and manipulate them. If you don't want all of the variables in your workspace, it is straightforward to add a line at the top to turn it into a function. If you don't want all the intermediate results printed, simply comment out the lines you don't want. We can find all information about code and conventional notation in [6].

The SCAM tool cannot simply read a schematic diagram so we need to develop a method for representing a circuit textually. This can be done using a device called a netlist that defines the interconnection between circuit elements. If you have used SPICE (Simulation Program with Integrated Circuit Emphasis) this is a familiar concept. SCAM requires a text file with one line for each component in the circuit. Each type of component has its own format for its corresponding lines in a file, as summarized in table 2. The labels N1, N2, etc., correspond to the nodes in the circuit [6].

Component Type	Symbol	SCAM Description
Resistor	$N1$ 1 kΩ R_1 $N2$	R1 N1 N2 1000 *R1 is between nodes N1 and N2, and has a value of 1000 Ohms *The value of the component must be written out (no abbreviations like kOhm) as a number. *The name of the component is Rx, where x can be any combination of letters and numbers. R1, Rabc, Ra1 are all valid names.
Capacitor	$N1$ 1 uF C_1 $N2$	C1 N1 N2 1E-6 *Similar comments to the resistor
Inductor	$N1$ 1 mH L_1 $N2$	L1 N1 N2 1E-3 *Similar comments to the resistor
Voltage Source	$N1$ I_{V_1} V_1 12 V $N2$	V1 N1 N2 12 *Similar comments to the resistor *Node N1 is connected to the positive node, N2 to the negative node. *The current through the source is one of the unknowns, it is defined as shown
Current Source	$N2$ I_1 $N1$	I1 N1 N2 1 *Similar comments to the resistor *Current flows out of node N1 and into node N2.
Op Amp	$N1$ + $N2$ − $N3$	O1 N1 N2 N3 *Similar comments to the resistor but with three nodes as shown.

Table 2: Component type format

The generic way to create the netlist file with the data required from the circuit to operate SCAM is shown in table 3 considering a resistor as an example, and having the option of giving a numerical value or leaving it symbolic in case you want to have an analytical result where values will be modified later.

Component	Node+	Node-	Numeric/Symbolic Value
R1	N1	N2	number or `symbolic`
⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮

Table 3: Netlist file format

Figure 6 shows a simple example for illustrative purposes. We start by defining the nodes. The only restriction here is that the nodes must be labeled such that ground is node 0, and the other nodes are named consecutively starting at 1. The choice of which number to assign to which node is entirely arbitrary.

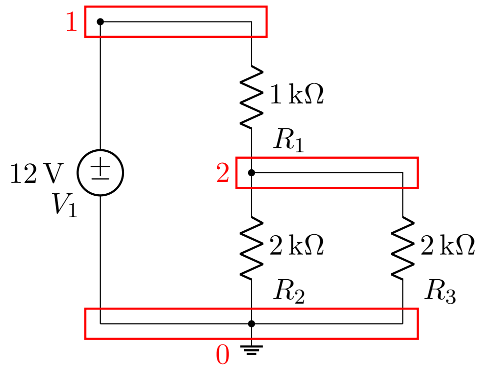


Figure 6: Example Circuit Nodes

The netlist file with *.cir* extension corresponding to the example in figure 6 is shown in table 4. The parameters of example circuit in figure 6 are

V1	1	0	12
R1	1	2	1000
R2	2	0	2000
R3	2	0	2000

Table 4: Netlist file

Parameter	**Symbol**	**Value**	**Unit**
Voltage	V_1	12	V
Resistor	R_1	1	kΩ
Resistor	R_2	2	kΩ
Resistor	R_3	2	kΩ

Table 5: Example Circuit Parameters

ilustrated in table 5.
Let's use the circuit in figure 6, we will create a text file containing the netlist in table 4 and save it in the directory seen by Matlab. I edited such a file (using the Matlab editor) and saved it in my SCAM directory as *example1.cir*. To run the program, assign the filename of the circuit to be analyzed to the variable fname, and then call the program. The output from the Matlab window is shown in figure 7.

```
>> fname="example1.cir";
>> scam

Started -- please be patient.

Netlist:
V1 1 0 12
R1 1 2 1000
R2 2 0 2000
R3 2 0 2000

The A matrix:
[  1/R1,               -1/R1, 1]
[ -1/R1, 1/R1 + 1/R2 + 1/R3, 0]
[     1,                   0, 0]

The x matrix:
  v_1
  v_2
 I_V1

The z matrix:
  0
  0
 V1

The matrix equation:
            I_V1 + v_1/R1 - v_2/R1 == 0
 v_2*(1/R1 + 1/R2 + 1/R3) - v_1/R1 == 0
                                v_1 == V1

The solution:
                                        v_1 == V1
       v_2 == (R2*R3*V1)/(R1*R2 + R1*R3 + R2*R3)
 I_V1 == -(V1*(R2 + R3))/(R1*R2 + R1*R3 + R2*R3)

Elapsed time is 0.308426 seconds.
```

Figure 7: SCAM tool process in Matlab's command window

We can use these variables shown in (2.28) (2.29) (2.30) to recreate the equations of the circuit in the form of (2.1), as it is shown in (2.27)

$$\begin{bmatrix} I\_\mathrm{V}_1 + \frac{v\_1}{R_1} - \frac{v\_2}{R_1} \\ \\ v\_2\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) - \frac{v\_1}{R_1} \\ \\ v\_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \mathrm{V}_1 \end{bmatrix} \tag{2.27}$$

In figure 7, the netlist is displayed, followed by A matrix, x vector and z vector as well as the matrix equations written out. Finally the values of the unknow variables are displayed.

$$A = \begin{bmatrix} \frac{1}{R_1} & -\frac{1}{R_1} & 1 \\ -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & 0 \\ 1 & 0 & 0 \end{bmatrix} \tag{2.28}$$

$$x = \begin{bmatrix} v\_1 \\ v\_2 \\ I\_\mathrm{V}_1 \end{bmatrix} \tag{2.29}$$

$$z = \begin{bmatrix} 0 \\ 0 \\ \mathrm{V}_1 \end{bmatrix} \tag{2.30}$$

As mentioned in section 2.1.1, we can see that each matrix represented in (2.28), (2.29) and (2.30), obtained algorithmically by SCAM, are the same as the matrices represented in expression (2.2) obtained analytically using the MNA method.

The solution of equations system presented by the SCAM tool in Matlab (2.31) is illustrated in the figure 7.

$$\mathbf{x} = \begin{bmatrix} v\_1 \\ v\_2 \\ I\_\mathrm{V}_1 \end{bmatrix} = \begin{bmatrix} V_1 \\ \frac{R_2\,R_3\,V_1}{R_1\,R_2+R_1\,R_3+R_2\,R_3} \\ -\frac{V_1\,(R_2+R_3)}{R_1\,R_2+R_1\,R_3+R_2\,R_3} \end{bmatrix} \tag{2.31}$$

The solution of equations system (2.31) obtained by SCAM is the same as solution (2.5) obtained analytically by applying MNA in the electrical circuit, therefore, the use of Matlab's SCAM tool for the analysis and resolution of equations systems in electrical circuits in a more fluid and efficient way is verified and confirmed.

Now, we can work with the expressions obtained through SCAM tool, like the value of v_2 (the voltage at node 2) through matlab command window, in this example,we have it in (2.31) and is shown in figure 8.

```
>> v_2
v_2 = (R2*R3*V1)/(R1*R2 + R1*R3 + R2*R3)
```

Figure 8: v_2 in Matlab command window

The value of I_V_1 (the current through the voltage source) we have it in (2.31) and is shown in figure 9.

```
>> I_V1
I_V1 = -(V1*(R2 + R3))/(R1*R2 + R1*R3 + R2*R3)
```

Figure 9: I_V_1 in Matlab command window

In addition to the unknowns, several other variables are created in the workspace (this is why the SCAM program is a script instead of a function). The important variables created, in addition to the unknowns, are a value corresponding to each of the elements. In figure 10 we can examine the value of any element, for example V_1 or R_1.

```
>> V1
V1 =    12

>> R1
R1 =    1000
```

Figure 10: Numeric value of V_1 and R_1 in Matlab command window

Figure 11 shows how we use these values from the workspace to obtain numerical values for the unknowns displayed in figure 8 and figure 9 respectively.

```
>> eval(v_2)
ans =     6

>> eval(I_V1)
ans =   -0.0060
```

Figure 11: Numeric values for the unknowns v_2 and I_V_1 in Matlab command window

Which shows that the voltage at node 2 is 6 volts, and the current through V_1 is 6 mA (into the positive node).

2.2 MNA method applied to MG of two single-phase inverters with a resistive load

The procedure to symbolically analyze the MG single-phase electrical model will follow the steps described in the section 2.1 referring to the example electrical circuit illustrated in figure 2 and obtain the system of equations as in the table 1 of each section of the electrical diagram.

Figure 12 shows the MG electrical scheme with two single-phase inverters and a resistive load to be considered.

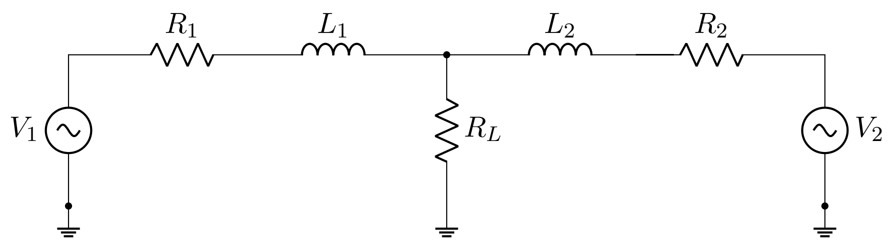


Figure 12: Microgrid single-phase electrical scheme

2.2.1 Manual analysis

From the scheme of figure 12, we identify the nodes and label them consecutively considering the reference node, as shown in figure 13.

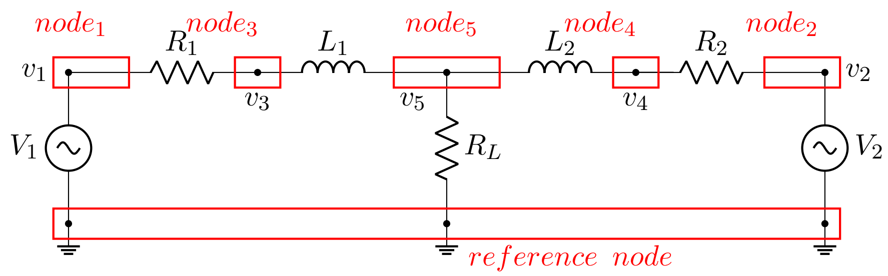


Figure 13: Microgrid single-phase electrical nodal scheme

The currents that pass through the voltage sources from the positive node to the negative node are defined in the figure 14.

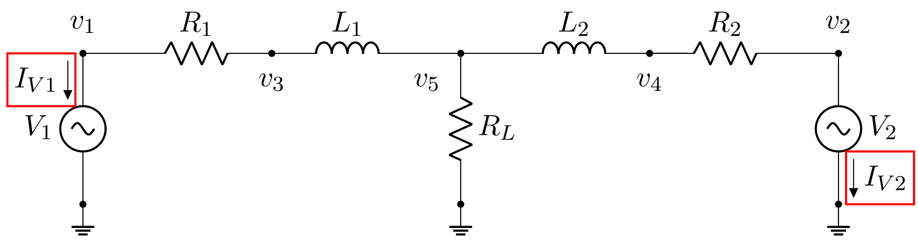


Figure 14: Currents through voltage sources in Microgrid single-phase electrical scheme

We apply Kirchhoff's currents law (**KCL**) in each node, considering as positive those that leave it, as shown in the figure 15. Then, we obtain the voltage equation of each voltage source and obtain the system of equations as illustrated in the table 6.

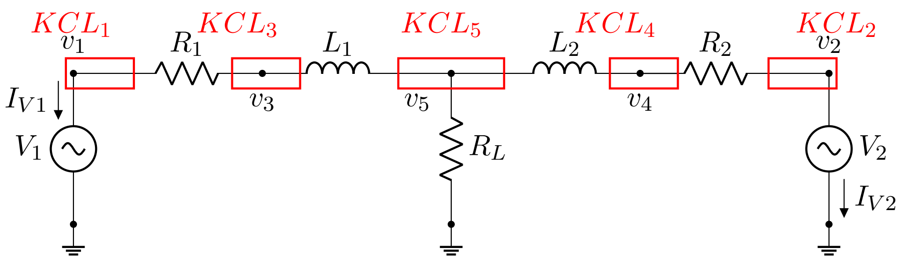


Figure 15: Kirchhoff's currents law in Microgrid single-phase electrical nodal scheme

The application of MNA to circuits with inductors and/or capacitors is very simple and practical if we use their complex impedance, where:

$$Z_R = R \tag{2.32}$$

$$Z_L = j\omega L = sL \tag{2.33}$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{sC} \tag{2.34}$$

In this case, as it is a resistive-inductive circuit, we will use the complex impedance in (2.32) and (2.33) respectively.

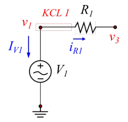
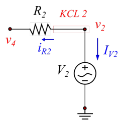
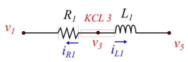
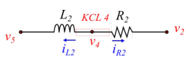
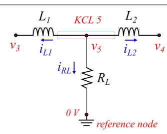
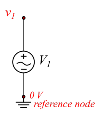
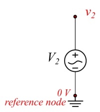
Electrical circuit section	Node equation	Matrix equation $\mathbf{Ax} = \mathbf{z}$
	$I_{V1} + i_{R_1} = 0 \Rightarrow I_{V1} + \frac{v_1 - v_3}{R_1} = 0 \Rightarrow \frac{v_1}{R_1} - \frac{v_3}{R_1} + I_{V1} = 0$	$\underbrace{\begin{bmatrix} \mathbf{\frac{1}{R_1}} & \mathbf{0} & -\mathbf{\frac{1}{R_1}} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ I_{V1} \\ I_{V2} \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} \mathbf{0} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \end{bmatrix}}_{\mathbf{z}}$
	$I_{V2} + i_{R_2} = 0 \Rightarrow I_{V2} + \frac{v_2 - v_4}{R_2} = 0 \Rightarrow \frac{v_2}{R_2} - \frac{v_4}{R_2} + I_{V2} = 0$	$\begin{bmatrix} \frac{1}{R_1} & 0 & -\frac{1}{R_1} & 0 & 0 & 1 & 0 \\ \mathbf{0} & \mathbf{\frac{1}{R_2}} & \mathbf{0} & -\mathbf{\frac{1}{R_2}} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ I_{V1} \\ I_{V2} \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{0} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \end{bmatrix}$
	$i_{R_1} + i_{L_1} = 0 \Rightarrow \frac{v_3 - v_1}{R_1} + \frac{v_3 - v_5}{sL_1} = 0 \Rightarrow -\frac{v_1}{R_1} + \left(\frac{1}{R_1} + \frac{1}{sL_1}\right) v_3 - \frac{v_5}{sL_1} = 0$	$\begin{bmatrix} \frac{1}{R_1} & 0 & -\frac{1}{R_1} & 0 & 0 & 1 & 0 \\ 0 & \frac{1}{R_2} & 0 & -\frac{1}{R_2} & 0 & 0 & 1 \\ -\mathbf{\frac{1}{R_1}} & \mathbf{0} & \left(\mathbf{\frac{1}{R_1}} + \mathbf{\frac{1}{sL_1}}\right) & \mathbf{0} & -\mathbf{\frac{1}{sL_1}} & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ I_{V1} \\ I_{V2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \mathbf{0} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \end{bmatrix}$
	$i_{R_2} + i_{L_2} = 0 \Rightarrow \frac{v_4 - v_2}{R_2} + \frac{v_4 - v_5}{sL_2} = 0 \Rightarrow -\frac{v_2}{R_2} + \left(\frac{1}{R_2} + \frac{1}{sL_2}\right) v_4 - \frac{v_5}{sL_2} = 0$	$\begin{bmatrix} \frac{1}{R_1} & 0 & -\frac{1}{R_1} & 0 & 0 & 1 & 0 \\ 0 & \frac{1}{R_2} & 0 & -\frac{1}{R_2} & 0 & 0 & 1 \\ -\frac{1}{R_1} & 0 & \left(\frac{1}{R_1} + \frac{1}{sL_1}\right) & 0 & -\frac{1}{sL_1} & 0 & 0 \\ \mathbf{0} & -\mathbf{\frac{1}{R_2}} & \mathbf{0} & \left(\mathbf{\frac{1}{R_2}} + \mathbf{\frac{1}{sL_2}}\right) & -\mathbf{\frac{1}{sL_2}} & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ I_{V1} \\ I_{V2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \mathbf{0} \\ \vdots \\ \vdots \\ \vdots \end{bmatrix}$
	$i_{L_1} + i_{L_2} + i_{R_L} = 0 \Rightarrow \frac{v_5 - v_3}{sL_1} + \frac{v_5 - v_4}{sL_2} + \frac{v_5}{R_L} = 0 \Rightarrow -\frac{v_3}{sL_1} - \frac{v_4}{sL_2} + \left(\frac{1}{sL_1} + \frac{1}{sL_2} + \frac{1}{R_L}\right) v_5 = 0$	$\begin{bmatrix} \frac{1}{R_1} & 0 & -\frac{1}{R_1} & 0 & 0 & 1 & 0 \\ 0 & \frac{1}{R_2} & 0 & -\frac{1}{R_2} & 0 & 0 & 1 \\ -\frac{1}{R_1} & 0 & \left(\frac{1}{R_1} + \frac{1}{sL_1}\right) & 0 & -\frac{1}{sL_1} & 0 & 0 \\ 0 & -\frac{1}{R_2} & 0 & \left(\frac{1}{R_2} + \frac{1}{sL_2}\right) & -\frac{1}{sL_2} & 0 & 0 \\ \mathbf{0} & \mathbf{0} & -\mathbf{\frac{1}{sL_1}} & -\mathbf{\frac{1}{sL_2}} & \left(\mathbf{\frac{1}{sL_1}} + \mathbf{\frac{1}{sL_2}} + \mathbf{\frac{1}{R_L}}\right) & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ I_{V1} \\ I_{V2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \mathbf{0} \\ \vdots \\ \vdots \end{bmatrix}$
	$v_1 - 0 = V_1 \Rightarrow v_1 = V_1$	$\begin{bmatrix} \frac{1}{R_1} & 0 & -\frac{1}{R_1} & 0 & 0 & 1 & 0 \\ 0 & \frac{1}{R_2} & 0 & -\frac{1}{R_2} & 0 & 0 & 1 \\ -\frac{1}{R_1} & 0 & \left(\frac{1}{R_1} + \frac{1}{sL_1}\right) & 0 & -\frac{1}{sL_1} & 0 & 0 \\ 0 & -\frac{1}{R_2} & 0 & \left(\frac{1}{R_2} + \frac{1}{sL_2}\right) & -\frac{1}{sL_2} & 0 & 0 \\ 0 & 0 & -\frac{1}{sL_1} & -\frac{1}{sL_2} & \left(\frac{1}{sL_1} + \frac{1}{sL_2} + \frac{1}{R_L}\right) & 0 & 0 \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ I_{V1} \\ I_{V2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \mathbf{V_1} \\ \vdots \end{bmatrix}$
	$v_2 - 0 = V_2 \Rightarrow v_2 = V_2$	$\begin{bmatrix} \frac{1}{R_1} & 0 & -\frac{1}{R_1} & 0 & 0 & 1 & 0 \\ 0 & \frac{1}{R_2} & 0 & -\frac{1}{R_2} & 0 & 0 & 1 \\ -\frac{1}{R_1} & 0 & \left(\frac{1}{R_1} + \frac{1}{sL_1}\right) & 0 & -\frac{1}{sL_1} & 0 & 0 \\ 0 & -\frac{1}{R_2} & 0 & \left(\frac{1}{R_2} + \frac{1}{sL_2}\right) & -\frac{1}{sL_2} & 0 & 0 \\ 0 & 0 & -\frac{1}{sL_1} & -\frac{1}{sL_2} & \left(\frac{1}{sL_1} + \frac{1}{sL_2} + \frac{1}{R_L}\right) & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ I_{V1} \\ I_{V2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V_1 \\ \mathbf{V_2} \end{bmatrix}$

Table 6: Process of obtaining the equations system of Microgrid single-phase electrical scheme

From the matrix equation (2.1) of the form $\mathbf{Ax} = \mathbf{z}$, we have the system of equations (2.35) shown in table 6.

$$\underbrace{\begin{bmatrix} \frac{1}{R_1} & 0 & -\frac{1}{R_1} & 0 & 0 & 1 & 0 \\ 0 & \frac{1}{R_2} & 0 & -\frac{1}{R_2} & 0 & 0 & 1 \\ -\frac{1}{R_1} & 0 & \left(\frac{1}{R_1} + \frac{1}{sL_1}\right) & 0 & -\frac{1}{sL_1} & 0 & 0 \\ 0 & -\frac{1}{R_2} & 0 & \left(\frac{1}{R_2} + \frac{1}{sL_2}\right) & -\frac{1}{sL_2} & 0 & 0 \\ 0 & 0 & -\frac{1}{sL_1} & -\frac{1}{sL_2} & \left(\frac{1}{sL_1} + \frac{1}{sL_2} + \frac{1}{R_L}\right) & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ I_{V1} \\ I_{V2} \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V_1 \\ V_2 \end{bmatrix}}_{\mathbf{z}} \quad (2.35)$$

The solution of the system of equations, using the matrix equation in (2.3) of the form $\mathbf{x} = \mathbf{A}^{-1}\mathbf{z}$ through Matlab's functions, is shown in (2.36)

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ I_{\mathrm{V1}} \\ I_{\mathrm{V2}} \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ \frac{V_1\left(R_2\,\mathrm{RL}+L_1\,R_2\,s+L_1\,\mathrm{RL}\,s+L_2\,\mathrm{RL}\,s+L_1\,L_2\,s^2\right)}{R_1\,R_2+R_1\,\mathrm{RL}+R_2\,\mathrm{RL}+L_1\,R_2\,s+L_2\,R_1\,s+L_1\,\mathrm{RL}\,s+L_2\,\mathrm{RL}\,s+L_1\,L_2\,s^2} + \frac{R_1\,\mathrm{RL}\,V_2}{R_1\,R_2+R_1\,\mathrm{RL}+R_2\,\mathrm{RL}+L_1\,R_2\,s+L_2\,R_1\,s+L_1\,\mathrm{RL}\,s+L_2\,\mathrm{RL}\,s+L_1\,L_2\,s^2} \\ \frac{V_2\left(R_1\,\mathrm{RL}+L_2\,R_1\,s+L_1\,\mathrm{RL}\,s+L_2\,\mathrm{RL}\,s+L_1\,L_2\,s^2\right)}{R_1\,R_2+R_1\,\mathrm{RL}+R_2\,\mathrm{RL}+L_1\,R_2\,s+L_2\,R_1\,s+L_1\,\mathrm{RL}\,s+L_2\,\mathrm{RL}\,s+L_1\,L_2\,s^2} + \frac{R_2\,\mathrm{RL}\,V_1}{R_1\,R_2+R_1\,\mathrm{RL}+R_2\,\mathrm{RL}+L_1\,R_2\,s+L_2\,R_1\,s+L_1\,\mathrm{RL}\,s+L_2\,\mathrm{RL}\,s+L_1\,L_2\,s^2} \\ \frac{V_2\,(R_1\,\mathrm{RL}+L_1\,\mathrm{RL}\,s)}{R_1\,R_2+R_1\,\mathrm{RL}+R_2\,\mathrm{RL}+L_1\,R_2\,s+L_2\,R_1\,s+L_1\,\mathrm{RL}\,s+L_2\,\mathrm{RL}\,s+L_1\,L_2\,s^2} + \frac{V_1\,(R_2\,\mathrm{RL}+L_2\,\mathrm{RL}\,s)}{R_1\,R_2+R_1\,\mathrm{RL}+R_2\,\mathrm{RL}+L_1\,R_2\,s+L_2\,R_1\,s+L_1\,\mathrm{RL}\,s+L_2\,\mathrm{RL}\,s+L_1\,L_2\,s^2} \\ \frac{\mathrm{RL}\,V_2}{R_1\,R_2+R_1\,\mathrm{RL}+R_2\,\mathrm{RL}+L_1\,R_2\,s+L_2\,R_1\,s+L_1\,\mathrm{RL}\,s+L_2\,\mathrm{RL}\,s+L_1\,L_2\,s^2} - \frac{V_1\,(R_2+\mathrm{RL}+L_2\,s)}{R_1\,R_2+R_1\,\mathrm{RL}+R_2\,\mathrm{RL}+L_1\,R_2\,s+L_2\,R_1\,s+L_1\,\mathrm{RL}\,s+L_2\,\mathrm{RL}\,s+L_1\,L_2\,s^2} \\ \frac{\mathrm{RL}\,V_1}{R_1\,R_2+R_1\,\mathrm{RL}+R_2\,\mathrm{RL}+L_1\,R_2\,s+L_2\,R_1\,s+L_1\,\mathrm{RL}\,s+L_2\,\mathrm{RL}\,s+L_1\,L_2\,s^2} - \frac{V_2\,(R_1+\mathrm{RL}+L_1\,s)}{R_1\,R_2+R_1\,\mathrm{RL}+R_2\,\mathrm{RL}+L_1\,R_2\,s+L_2\,R_1\,s+L_1\,\mathrm{RL}\,s+L_2\,\mathrm{RL}\,s+L_1\,L_2\,s^2} \end{bmatrix} \tag{2.36}$$

2.2.2 SCAM computer analysis

The process for the equations nodal analysis of the MG single-phase electrical model using SCAM tool in Matlab will follow the steps described in the section 2.1.3 referring to the electrical diagram of the figure 12, as shown in figure 16.

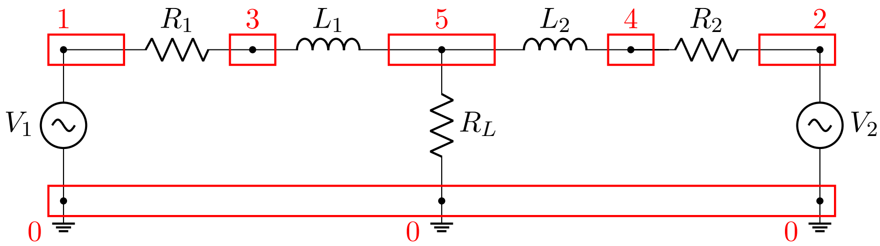


Figure 16: Definition of nodes for SCAM netlist in Microgrid single-phase electrical scheme

The netlist file with *.cir* extension corresponding to MG single-phase electrical model in figure 12 is shown in table 7. The netlist column where the expression *symbolic* is represented instead of numerical values, allows us to later assign the values to the variables related to the components of the electrical circuit, when we want to carry out tests with different values directly from MATLAB editor or command window, without modifying the file with a *.cir* extension to carry out this task.

V1	1	0	symbolic
V2	2	0	symbolic
L1	3	5	symbolic
L2	4	5	symbolic
R1	1	3	symbolic
R2	2	4	symbolic
RL	5	0	symbolic

Table 7: Netlist file of MG single-phase electrical nodal scheme

We can use these variables shown in (2.38) (2.39) (2.40) to recreate the equations of the circuit in the form of (2.1), as it is shown in (2.37)

$$\begin{bmatrix} I_{\mathrm{V1}} + \frac{v_1}{R_1} - \frac{v_3}{R_1} \\ I_{\mathrm{V2}} + \frac{v_2}{R_2} - \frac{v_4}{R_2} \\ v_3 \left(\frac{1}{R_1} + \frac{1}{L_1\,s}\right) - \frac{v_1}{R_1} - \frac{v_5}{L_1\,s} \\ v_4 \left(\frac{1}{R_2} + \frac{1}{L_2\,s}\right) - \frac{v_2}{R_2} - \frac{v_5}{L_2\,s} \\ v_5 \left(\frac{1}{\mathrm{RL}} + \frac{1}{L_1\,s} + \frac{1}{L_2\,s}\right) - \frac{v_3}{L_1\,s} - \frac{v_4}{L_2\,s} \\ v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V_1 \\ V_2 \end{bmatrix} \tag{2.37}$$

From the data entered in the netlist file of table 7 corresponding to the circuit of figure 12, the SCAM tool gives us the values of the A matrix (2.38), x vector (2.39) and z vector (2.40), followed by the approach (2.37) and solving the system of equations using the MNA method.

$$A = \begin{bmatrix} \frac{1}{R_1} & 0 & -\frac{1}{R_1} & 0 & 0 & 1 & 0 \\ 0 & \frac{1}{R_2} & 0 & -\frac{1}{R_2} & 0 & 0 & 1 \\ -\frac{1}{R_1} & 0 & \frac{1}{R_1} + \frac{1}{L_1\,s} & 0 & -\frac{1}{L_1\,s} & 0 & 0 \\ 0 & -\frac{1}{R_2} & 0 & \frac{1}{R_2} + \frac{1}{L_2\,s} & -\frac{1}{L_2\,s} & 0 & 0 \\ 0 & 0 & -\frac{1}{L_1\,s} & -\frac{1}{L_2\,s} & \frac{1}{\mathrm{RL}} + \frac{1}{L_1\,s} + \frac{1}{L_2\,s} & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{2.38}$$

$$x = \begin{bmatrix} v_1 & v_2 & v_3 & v_4 & v_5 & I_{\mathrm{V1}} & I_{\mathrm{V2}} \end{bmatrix}^T \tag{2.39}$$

$$z = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & V_1 & V_2 \end{bmatrix}^T \tag{2.40}$$

2.3 Transfer Function (TF) model to State Space (SS) model from MNA solution

The solution obtained with SCAM is given by the matricial expression in (2.3) as $\mathbf{x} = \mathbf{A^{-1}z}$ the same that is translated in the expression (2.41), where the matrices $\mathbf{z} \equiv U(s)$ are the system inputs corresponding to the inverter voltages, the matrices $\mathbf{A^{-1}} \equiv G(s)$ are the transfer function of the system and the matrices $\mathbf{x} \equiv Y(s)$ are the outputs corresponding to the currents that pass through each inverter in Multiple Input Multiple Output (MIMO) systems. Information about modeling in state space can be found in [8].

The general expression of the transfer function matrix in a multivariate system is given as follows

$$
\begin{aligned}
Y(s) =& G(s)U(s) && (2.41)\\
=& \left[C\left(sI-A\right)^{-1}B+D\right]U(s) && (2.42)
\end{aligned}
$$

$$
\Rightarrow \begin{bmatrix} Y_1(s) \\ \vdots \\ Y_q(s) \end{bmatrix} = \begin{bmatrix} \frac{n_{11}(s)}{d_{11}(s)} & \cdots & \frac{n_{1p}(s)}{d_{1p}(s)} \\ \vdots & \cdots & \vdots \\ \frac{n_{q1}(s)}{d_{q1}(s)} & \cdots & \frac{n_{qp}(s)}{d_{qp}(s)} \end{bmatrix} \begin{bmatrix} U_1(s) \\ \vdots \\ U_p(s) \end{bmatrix} \tag{2.43}
$$

To understand how to go from transfer function to state space in general in a MIMO system [12], we take as an example a transfer function matrix with two inputs and two outputs as follows

$$
\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{1}{(s+a)(s+b)} & \frac{1}{(s+b)(s+d)} \\ \frac{1}{(s+a)(s+c)} & \frac{1}{(s+c)(s+d)} \end{bmatrix}}_{G(s)} \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix} \tag{2.44}
$$

We treat the MIMO system (2.44) as two MISO systems with respect to each output Y_1 and Y_2 after obtaining the least common polynomial denominator $\frac{1}{(s+b)}$ and $\frac{1}{(s+c)}$ respectively, and the state variables are defined as follows

$$
\begin{array}{l|l}
Z_{11}(s) \triangleq \frac{U_1(s)}{(s+a)} & Z_{21}(s) \triangleq \frac{U_2(s)}{(s+d)} \\
\dot{z_{11}} + az_{11} = u_1(t) & \dot{z_{21}} + dz_{21} = u_2(t) \\
x_1 \triangleq z_{11} & x_2 \triangleq z_{21} \\
\dot{x_1} = -ax_1 + u_1(t) & \dot{x_2} = -dx_2 + u_2(t)
\end{array} \tag{2.45}
$$

With the state variables defined in (2.45) and the least common denominator obtained above, we can write the matrix of the system's transfer function as

$$
\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} \frac{z_{11}(s)}{(s+b)} + \frac{z_{21}(s)}{(s+b)} \\ \frac{z_{11}(s)}{(s+c)} + \frac{z_{21}(s)}{(s+c)} \end{bmatrix} \tag{2.46}
$$

From the standard canonical form of SISO systems [13], the system in (2.46) can be viewed as two separate two-input and one-output systems with the state-space representation as follow

$$
\begin{array}{l|l}
\dot{y_1} + by_1 = z_{11} + z_{21} & \dot{y_2} + cy_2 = z_{11} + z_{21} \\
x_3 \triangleq y_1 & x_4 \triangleq y_2 \\
\dot{x_3} = -bx_3 + x_1 + x_2 & \dot{x_4} = -cx_4 + x_1 + x_2
\end{array} \tag{2.47}
$$

Finally, we have the matrix representation of the system in state space by grouping all the differential equations involving the state variables, as follows in (2.48) (2.49) whose order state four is the minimum verifiable in [14] under the parameters of this example.

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \end{bmatrix} = \underbrace{\begin{bmatrix} -a & 0 & 0 & 0 \\ 0 & -d & 0 & 0 \\ 1 & 1 & -b & 0 \\ 1 & 1 & 0 & -c \end{bmatrix}}_{\mathbf{A}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}_{\mathbf{B}} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{2.48}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\mathbf{C}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \tag{2.49}$$

In order to express the internal representation, two matrix equations will be needed: the state equation (2.50) and the output equation (2.51).

$$\dot{x}(t) = A\,x(t) + B\,u(t) \tag{2.50}$$

$$y(t) = C\,x(t) + D\,u(t) \tag{2.51}$$

where:
A: Status matrix
B: Input matrix
C: Output matrix
D: Direct input-output coupling matrix

Before going on to see how these matrices can be obtained, the case of multivariable continuous systems will be considered, where it is considered that the system has p inputs and q outputs, known as the MIMO system (Multiple inputs Multiple outputs) represented in the figure 17 :

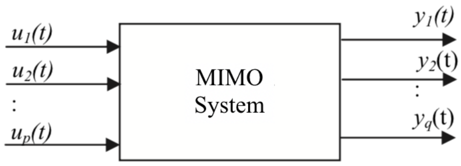


Figure 17: Multiple Input Multiple Output System

The approach for these systems is to work with n first order equations, where you do not have a single input and output, but a vector with the inputs (2.52) and another with the outputs (2.53) of system:

$$u(t) = \begin{pmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_p(t) \end{pmatrix} \tag{2.52}$$

$$y(t) = \begin{pmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_q(t) \end{pmatrix} \tag{2.53}$$

Expressed in matrix form, what is known as the state equation (2.54) and the output equation (2.55) is obtained.

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \vdots \\ \dot{x}_n(t) \end{pmatrix} = \underbrace{\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix}}_{\mathbf{A}} \begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{pmatrix} + \underbrace{\begin{bmatrix} b_{11} & b_{12} & \dots & b_{1p} \\ b_{21} & b_{22} & \dots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{np} \end{bmatrix}}_{\mathbf{B}} \begin{pmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_p(t) \end{pmatrix} \tag{2.54}$$

$$\begin{pmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_q(t) \end{pmatrix} = \underbrace{\begin{bmatrix} c_{11} & c_{12} & \dots & c_{1n} \\ c_{21} & c_{22} & \dots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{q1} & c_{q2} & \dots & c_{qn} \end{bmatrix}}_{\mathbf{C}} \begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{pmatrix} \tag{2.55}$$

As can be seen, in this case B will be the input matrix of dimensions $n \times p$ and C the matrix $q \times n$ of output of the system.

2.3.1 Computer analysis applied to MG of two single-phase inverters with a resistive load

From the solution obtained by SCAM, the section that interests me includes the equations that relate the currents as a function of the voltages expressed in (2.43) that was explained in section 2.3, and I obtain the matrix of transfer functions that relates each output to each input of the system using Matlab functions. To obtain the transfer function (2.56) it is necessary to know that the inputs of the electrical circuit correspond to the voltage and the outputs thereof corresponding to the currents in each independent voltage source. In this work, two single-phase sources V_1 and V_2 are presented, and as outputs of the system we have currents I_{V1} and I_{V2} respectively.

$$G_{i,i}(s) = \frac{Y(s)}{U(s)} = \frac{I_{\mathrm{V}_i}(s)}{V_i(s)}, \quad i = 1,2 \tag{2.56}$$

With the values obtained from the SCAM tool and stored in the workspace, we extract the output variables I_{V1} (2.57) and I_{V2} (2.59), each based on inputs V_1 and V_2.

$$I_{\mathrm{V1}}(s) = \frac{R_2\,V_1 + R_L\,V_1 - R_L\,V_2 + L_2\,V_1\,s}{R_1\,R_2 + R_1\,R_L + R_2\,R_L + L_1\,R_2\,s + L_2\,R_1\,s + L_1\,R_L\,s + L_2\,R_L\,s + L_1\,L_2\,s^2} \tag{2.57}$$

$$\begin{aligned}\Rightarrow I_{\mathrm{V1}}(s) = &\left(\frac{R_2 + R_L + L_2\,s}{R_1\,R_2 + R_1\,R_L + R_2\,R_L + L_1\,R_2\,s + L_2\,R_1\,s + L_1\,R_L\,s + L_2\,R_L\,s + L_1\,L_2\,s^2}\right) V_1 \\ &+ \left(\frac{-R_L}{R_1\,R_2 + R_1\,R_L + R_2\,R_L + L_1\,R_2\,s + L_2\,R_1\,s + L_1\,R_L\,s + L_2\,R_L\,s + L_1\,L_2\,s^2}\right) V_2\end{aligned} \tag{2.58}$$

In order to more easily recognize the coefficients of each input variable V_1 and V_2, we make use of the `coeffs` command, obtaining better equations distributed in (2.58) and (2.60).

$$I_{\mathrm{V2}}(s) = \frac{R_1\,V_2 - R_L\,V_1 + R_L\,V_2 + L_1\,V_2\,s}{R_1\,R_2 + R_1\,R_L + R_2\,R_L + L_1\,R_2\,s + L_2\,R_1\,s + L_1\,R_L\,s + L_2\,R_L\,s + L_1\,L_2\,s^2} \tag{2.59}$$

$$\begin{aligned}\Rightarrow I_{\mathrm{V2}}(s) = &\left(\frac{-R_L}{R_1\,R_2 + R_1\,R_L + R_2\,R_L + L_1\,R_2\,s + L_2\,R_1\,s + L_1\,R_L\,s + L_2\,R_L\,s + L_1\,L_2\,s^2}\right) V_1 \\ &+ \left(\frac{R_1 + R_L + L_1\,s}{R_1\,R_2 + R_1\,R_L + R_2\,R_L + L_1\,R_2\,s + L_2\,R_1\,s + L_1\,R_L\,s + L_2\,R_L\,s + L_1\,L_2\,s^2}\right) V_2\end{aligned} \tag{2.60}$$

In this work we have a system with two inputs and two outputs, in which each output depends on the two inputs, therefore, we will have a 2×2 matrix with four transfer functions corresponding to (2.61).

$$G(s) = \begin{bmatrix} G_{1,1}(s) & G_{1,2}(s) \\ G_{2,1}(s) & G_{2,2}(s) \end{bmatrix} \tag{2.61}$$

Starting from (2.58) and (2.60), each component as a transfer function corresponding to each output depending on each input, is detailed in (2.62) (2.63) (2.64) (2.65).

$$G_{1,1}(s) = \frac{I_{\mathrm{V1}}(s)}{V_1(s)} = \left(\frac{R_2 + R_L + L_2\,s}{R_1\,R_2 + R_1\,R_L + R_2\,R_L + L_1\,R_2\,s + L_2\,R_1\,s + L_1\,R_L\,s + L_2\,R_L\,s + L_1\,L_2\,s^2}\right) \tag{2.62}$$

$$G_{1,2}(s) = \frac{I_{\mathrm{V1}}(s)}{V_2(s)} = \left(\frac{-R_L}{R_1\,R_2 + R_1\,R_L + R_2\,R_L + L_1\,R_2\,s + L_2\,R_1\,s + L_1\,R_L\,s + L_2\,R_L\,s + L_1\,L_2\,s^2}\right) \tag{2.63}$$

$$G_{2,1}(s) = \frac{I_{\mathrm{V2}}(s)}{V_1(s)} = \left(\frac{-R_L}{R_1\,R_2 + R_1\,R_L + R_2\,R_L + L_1\,R_2\,s + L_2\,R_1\,s + L_1\,R_L\,s + L_2\,R_L\,s + L_1\,L_2\,s^2}\right) \tag{2.64}$$

$$G_{2,2}(s) = \frac{I_{V2}(s)}{V_2(s)} = \left(\frac{R_1 + R_L + L_1\,s}{R_1\,R_2 + R_1\,R_L + R_2\,R_L + L_1\,R_2\,s + L_2\,R_1\,s + L_1\,R_L\,s + L_2\,R_L\,s + L_1\,L_2\,s^2} \right) \tag{2.65}$$

$$G(s) = \begin{bmatrix} \frac{R_2+\mathrm{RL}+L_2\,s}{R_1\,R_2+R_1\,\mathrm{RL}+R_2\,\mathrm{RL}+L_1\,R_2\,s+L_2\,R_1\,s+L_1\,\mathrm{RL}\,s+L_2\,\mathrm{RL}\,s+L_1\,L_2\,s^2} & -\frac{\mathrm{RL}}{R_1\,R_2+R_1\,\mathrm{RL}+R_2\,\mathrm{RL}+L_1\,R_2\,s+L_2\,R_1\,s+L_1\,\mathrm{RL}\,s+L_2\,\mathrm{RL}\,s+L_1\,L_2\,s^2} \\ -\frac{\mathrm{RL}}{R_1\,R_2+R_1\,\mathrm{RL}+R_2\,\mathrm{RL}+L_1\,R_2\,s+L_2\,R_1\,s+L_1\,\mathrm{RL}\,s+L_2\,\mathrm{RL}\,s+L_1\,L_2\,s^2} & \frac{R_1+\mathrm{RL}+L_1\,s}{R_1\,R_2+R_1\,\mathrm{RL}+R_2\,\mathrm{RL}+L_1\,R_2\,s+L_2\,R_1\,s+L_1\,\mathrm{RL}\,s+L_2\,\mathrm{RL}\,s+L_1\,L_2\,s^2} \end{bmatrix} \tag{2.66}$$

Parameter	**Symbol**	**Value**	**Unit**
Inversor 1	V_1	155	V_p
Inversor 2	V_2	155	V_p
Resistor 1	R_1	3	Ω
Resistor 2	R_2	3	Ω
Resistor Load	R_L	1	kΩ
Inductor 1	L_1	10	mH
Inductor 2	L_2	10	mH

Table 8: MG single-phase model parameters

To obtain the results with numerical values, we use the `subs` command in MATLAB considering the MG parameters shown in table 8, achieving a 2×2 matrix of four transfer functions (2.67) as follow.

$$\Rightarrow G(s) = \begin{bmatrix} \frac{\frac{s}{100} + 1003}{\frac{s^2}{10000} + \frac{1003\,s}{50} + 6009} & -\frac{1000}{\frac{s^2}{10000} + \frac{1003\,s}{50} + 6009} \\ -\frac{1000}{\frac{s^2}{10000} + \frac{1003\,s}{50} + 6009} & \frac{\frac{s}{100} + 1003}{\frac{s^2}{10000} + \frac{1003\,s}{50} + 6009} \end{bmatrix} \tag{2.67}$$

We now proceed to perform the system analysis on a state space model. To express the state of the system, matrices and vectors will be used as a working tool, which is very suitable for expressing calculations and operations in computational terms.

To obtain matrices A (2.69), B (2.70), C (2.71) and D (2.72), from the transfer matrix $G(s)$ of the whole system that groups the two outputs based on the two inputs (2.67), we use the script, shown below (listing 2), to convert it in an object type `tf` (transfer function $\mathtt{G_{tf}}$), and this in turn, transform into an object type `ss` (state space $\mathtt{G_{ss}}$) with the command of the same name (2.68), where we can practically obtain the required matrices to continue with the analysis of this work in a state space model.

$$\mathtt{G_{ss}} = \mathtt{ss}\left(\mathtt{G_{tf}}\right) \tag{2.68}$$

```
s = tf('s'); % Make Generic Transfer Function Simple
[rows,columns]=size(G); % Rows and Columns of G
   Matrix (Various Func. Transf)
Gtf = tf(zeros(size(G)));
for fil=1:rows
  for col=1:columns
  Gtf(fil,col)=eval(char(G(fil,col)));
  end
end
```

Algorithm 2: Transfer function Matlab script

Once we have the model in state space in ss format, we can extract the model components that refer to the matrices A (2.69), B (2.70), C (2.71) and D (2.72) respectively.

$$A = G_{ss}.A = \begin{bmatrix} -2.006e5 & -7335 & 0 & 0 \\ 8192 & 0 & 0 & 0 \\ 0 & 0 & -2.006e5 & -7335 \\ 0 & 0 & 8192 & 0 \end{bmatrix} \tag{2.69}$$

$$B = G_{ss}.B = \begin{bmatrix} 64 & 0 \\ 0 & 0 \\ 0 & 64 \\ 0 & 0 \end{bmatrix} \tag{2.70}$$

$$C = G_{ss}.C = \begin{bmatrix} 1.562 & 19.13 & 0 & -19.07 \\ 0 & -19.07 & 1.562 & 19.13 \end{bmatrix} \tag{2.71}$$

$$D = G_{ss}.D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{2.72}$$

2.3.2 Simulation of MG of two single-phase inverters with a resistive load

The simulation of the MG single-phase electrical model has been performed with a Matlab script using Euler (2.75) for the states equation (2.50) and the outputs equation (2.51), where the matrices A (2.69), B (2.70), C (2.71) and D (2.72) obtained in the previous section are used .

A simulation time $\mathtt{t_f} = \mathtt{0.1}$ seconds has been configured, with steps $\mathtt{t_s} = 1\mathrm{e} - 6$, a frequency $\omega = 2\pi\mathtt{f} = 2\pi(60)$. In this work, we assume a phase of the first voltage source $\varphi_1 = 0$ and a range of values for the phase of the second voltage source $\varphi_2 = 0 : 0.0001 : 0.001$ in order to compare the behavior of the system when the two sources gradually phase out of each other and observe the voltage, current and power signals graphically.

We use input one (2.73) and input two (2.74)of voltage as a function of a sine:

$$\mathtt{u_1(t)} = \mathtt{u_0}\,\sin\left(\omega\,\mathtt{t} + \varphi_1\right), \quad \mathtt{u_0} = \mathtt{155} \tag{2.73}$$

$$\mathtt{u_2(t)} = \mathtt{u_0}\,\sin\left(\omega\,\mathtt{t} + \varphi_2\right), \quad \mathtt{u_0} = \mathtt{155} \tag{2.74}$$

To perform this simulation, we use Euler in the states equation (2.50) as shown in (2.75):

$$\begin{aligned} \mathtt{x(i+1)} &= \mathtt{x(i)} + \mathtt{t_s}\,\mathtt{x} \\ \Rightarrow \mathtt{x(i+1)} &= \mathtt{x(i)} + \mathtt{t_s}\left(\mathtt{A}\,\mathtt{x(i)} + \mathtt{B}\,\mathtt{u}\right), \quad \mathtt{i} = \mathtt{0} : \mathtt{length}\left(\frac{\mathtt{t_f}}{\mathtt{t_s}}\right) \end{aligned} \tag{2.75}$$

And the output equation (2.51), in this example, is given by (2.76);

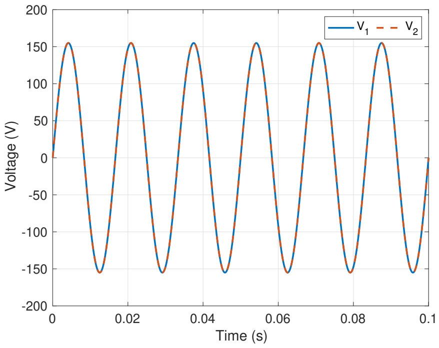
$$\mathtt{I} = \mathtt{C}\,\mathtt{x} \tag{2.76}$$

Because the values we obtain for voltage and current are stored in a vector format according to the number of inputs and outputs of the system, it is simple to obtain the power with the product of these two variables. Similarly, to calculate the rms power, you only need to apply the `rms` command to the power and save it vectorially to make the graphs.

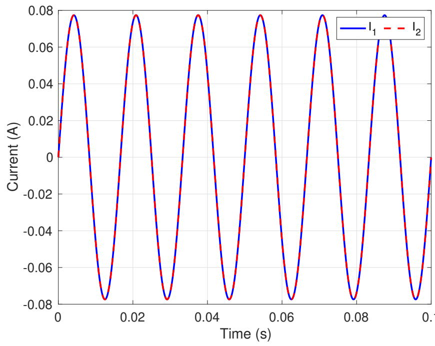
Using Matlab script (Section 7.1.1), we obtain the graph corresponding to current, voltage and power of each source. We have carried out the test with four options:

- Using two input sources with equal voltages and same phase.
- Using two input sources with different voltages and same phase.
- Using two input sources with equal voltages, but out of phase.
- Using two input sources with different voltages, but out of phase.

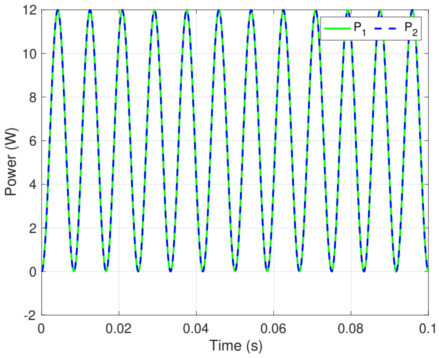
In figure 18, we observe the dynamics of the system referring to the voltage signal (subfigure 18a), the current signal (subfigure 18b), the active power signal (subfigure 18c) and the rms power signal (subfigure 18d) of the two sources of equal magnitude and phase during the established simulation time.



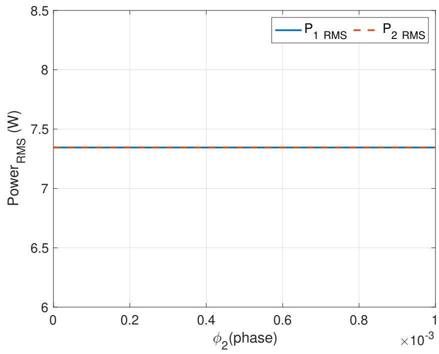
(a) Voltage



(b) Current



(c) Active Power

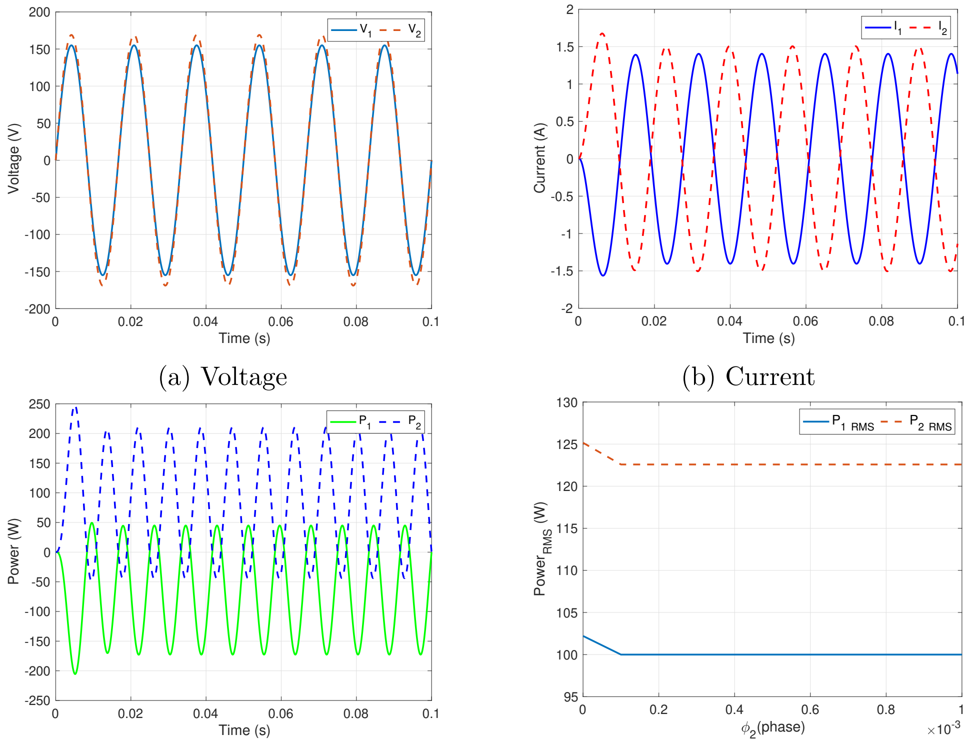


(d) RMS Active Power

Figure 18: System dynamics when two input sources with equal voltages and equal phases operate during the simulation time.

The graph in subfigure 18d corresponds to the rms power of each source as a function of phase φ_2 of the second. In this it can be seen that both sources deliver the same power, therefore, the rms power remains constant.

We observe in figure 19 the dynamics of the system regarding the voltage signal (subfigure 19a), the current signal (subfigure 19b), the active power signal (subfigure 19c) and the rms power signal (subfigure 19d) of the two sources of different magnitude and of the same phase during the established simulation time.



(a) Voltage (b) Current

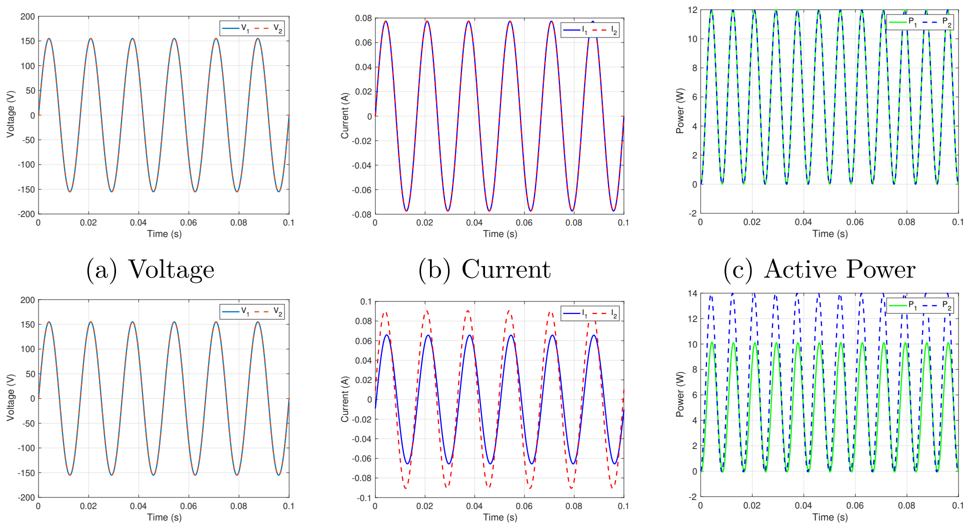
(c) Active Power (d) RMS Active Power

Figure 19: System dynamics when two input sources with different voltages and equal phases operate during the simulation time.

In the subfigure 19a, the signal of V_1 with a peak voltage of 155 volts and V_2 with a peak voltage of 169 volts are shown, keeping the phases the same. By having the different voltages, there is an increase in current and a phase shift between the two sources as seen in subfigure 19b. The currents of greater magnitude in subfigure 19c produce powers of considerable magnitude with respect to the powers displayed in subfigure 18c. The subfigure 19d corresponds to the rms power of each source as a function of phase φ_2 of the second. In this it can be seen that both sources deliver different powers but they remain constant during the simulation time.

Figure 20 shows the dynamics of the system with respect to the voltage signal (subfigure 20a), the current signal (subfigure 20b) and the active power signal (subfigure 20c) when the simulation starts with the phases $\varphi_1 = \varphi_2 = 0$. Then, it shows the dynamics of the system with respect to the voltage signal (subfigure 20d), the current signal (subfigure 20e) and the active power signal (subfigure 20f) when the simulation ends with the phases $\varphi_1 = 0$ and $\varphi_2 = 0.001$. The phase difference between φ_1 and φ_2 of the source voltage signal is programmed in Matlab in a range of 0 to 0.001

with phase increment steps of 0.0001 in φ_2 to illustrate the behavior of these signals.



(a) Voltage (b) Current (c) Active Power

(d) Voltage (e) Current (f) Active Power

Figure 20: System dynamics when two input sources with equal voltages and different phases operate during the simulation time.

The out of phase effect of the voltage of the second source is evident in the current I_2 and I_1 shown in the subfigure 20e, since there is an increase and decrease in their magnitudes respectively, in addition to the phase difference between these, which It also causes a similar effect on the power P_1 and P_2 appreciable in subfigure 20f.

The rms power of each source as a function of phase φ_2 of the second is shown in figure 21 . The rms power delivered by each source varies depending on the phase between them, that is, when the phase φ_2 is increased by a rate of 0.0001 and the phase φ_1 remains fixed, therefore, the power P_2 of the second source increases while the power P_1 of the first source decreases in the same proportion.

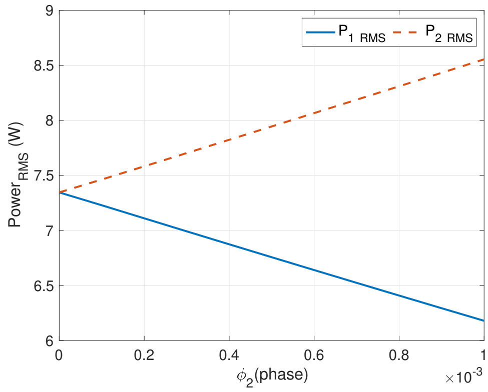
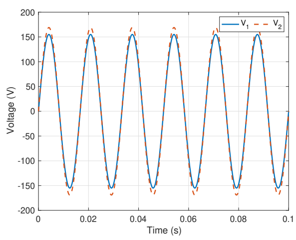
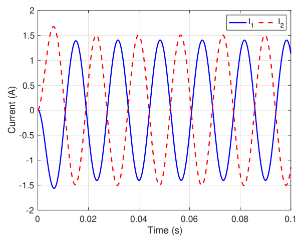


Figure 21: RMS Active Power

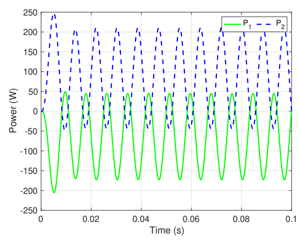
Figure 22 shows the dynamics of the system with respect to the voltage signal (subfigure 22a), the current signal (subfigure 22b) and the signal active power (subfigure 22c) when the simulation starts with phases $\varphi_1 = \varphi_2 = 0$. Next, it shows the system dynamics with respect to the voltage signal (subfigure 22d), the current signal (subfigure 20e) and the active power signal (subfigure 22f) when the simulation ends with phases $\varphi_1 = 0$ and $\varphi_2 = 0.001$ to illustrate the behavior of these signals.



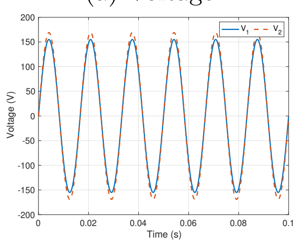
(a) Voltage



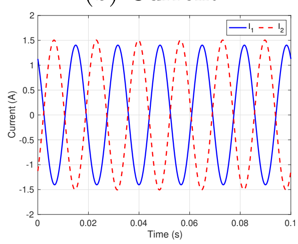
(b) Current



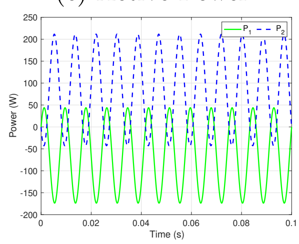
(c) Active Power



(d) Voltage



(e) Current



(f) Active Power

Figure 22: System dynamics when two input sources with different voltages and different phases operate during the simulation time.

The magnitude and out of phase effect of the voltage of the second source is evident in the current I_2 and I_1 shown in the subfigure 22e, since there is an increase and decrease in their magnitudes respectively, in addition to a significant phase difference phase between these, which It also causes a similar effect on the power P_1 and P_2 appreciable in subfigure 22c and subfigure 22f.

The rms power of each source as a function of phase φ_2 of the second is shown in figure 23 . The rms power delivered by each source varies depending on the phase between them, that is, when the phase φ_2 is increased by a rate of 0.0001 and the phase φ_1 remains fixed, therefore, the powers P_2 and P_1 have a behavior similar to that of the subfigure 19d, where the sources deliver a different power to each other constantly but with a slight increase while the programmed out of phase occurs during the simulation time.

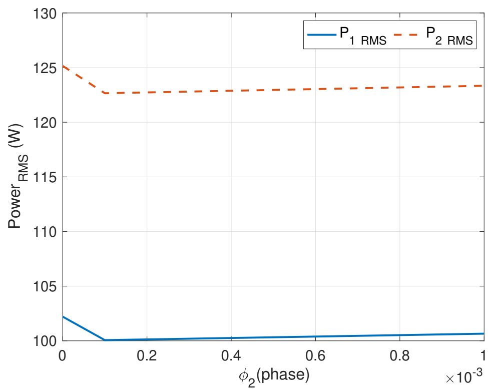


Figure 23: RMS Active Power

2.4 Pros and Cons from TF model to SS model since MNA solution

By applying the transformation of the transfer function model to the state space model of the solution obtained through SCAM, we have the Pros as follows

- The power is obtained directly with the product of the current and the voltage, because in the transfer function model we would have to apply the integral of the convolution of the product of the current and the voltage, and this makes the analysis become more difficult to analyze and automate using algorithms in Matlab.
- A better perception of the interaction of the components of the MG is achieved through the matrices A, B, C and D of state space model, which is very easy to apply in Matlab through functions known as `tf` and `ss` .

Among the Cons that can be highlighted we have:

- A considerable dimension of the matrices A, B, C and D in the state space model when the MG gets bigger and bigger, i.e. when it is composed of a significant number of investors.

- As a result of the point mentioned above, the intuition and facolity of analysis that is carried out with MGs of a larger volume of investors would be widely overshadowed, which makes it difficult for us to understand the behavior of the system in better detail. This situation is not observed in the case of the MG of two single-phase inverters with a resistive load, but in the case of the MG of two three-phase inverters with a resistive load we can clearly notice it.

3 MNA-based modeling and analysis of MG of two three-phase inverters with a resistive load

In the previous sections, methods of analysis of systems of equations related to electrical circuits such as MNA have been applied manually and in an automated way through SCAM in Matlab, and we have been able to verify in each section that the results obtained with these methods are The same and therefore, the modeling of MGs using automated algorithms in Matlab are perfectly applicable and scalable to various inverters, reducing the time involved in a manual analysis of circuits of this size.

3.1 Electrical scheme of three-phase MG

Said the above, we are going to analyze a MG with two three-phase inverters (V_1a, V_1b, V_1c and V_2a, V_2b, V_2c) with a pure resistive star load (R_La, R_Lb, R_Lc), considering that the lines or nodes that interconnect the MG inverters have resistive-inductive ($R_{1a,1b,1c}$, $L_{1a,1b,1c}$) and ($R_{2a,2b,2c}$, $L_{2a,2b,2c}$) characteristics. Figure 24 shows the MG electric scheme being considered with the respective nodes identified for the creation of the netlist file that SCAM will use in its execution.

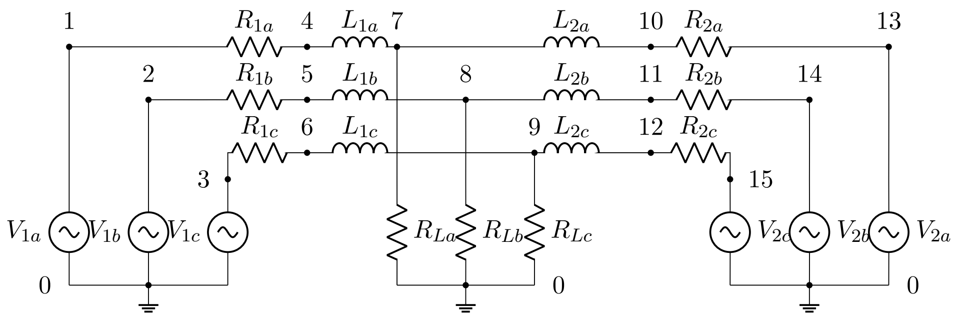


Figure 24: Microgrid three-phase electrical nodal scheme

3.2 SCAM computer analysis of equations

The netlist file with *.cir* extension corresponding to MG three-phase electrical model in figure 24 is shown in table 9. The netlist column where the expression *symbolic* is represented instead of numerical values, allows us to later assign the values to the variables related to the components of the electrical circuit, when we want to carry out tests with different values directly from MATLAB editor or command window, without modifying the file with a *.cir* extension to carry out this task.

V1a	1	0	Symbolic
V1b	2	0	Symbolic
V1c	3	0	Symbolic
V2a	13	0	Symbolic
V2b	14	0	Symbolic
V2c	15	0	Symbolic
L1a	4	7	Symbolic
L1b	5	8	Symbolic
L1c	6	9	Symbolic
L2a	10	7	Symbolic
L2b	11	8	Symbolic
L2c	12	9	Symbolic
R1a	1	4	Symbolic
R1b	2	5	Symbolic
R1c	3	6	Symbolic
R2a	13	10	Symbolic
R2b	14	11	Symbolic
R2c	15	12	Symbolic
RLa	7	0	Symbolic
RLb	8	0	Symbolic
RLc	9	0	Symbolic

Table 9: Netlist file of MG three-phase electrical nodal scheme

From the data entered in the netlist file of table 9 corresponding to the circuit of figure 24, the SCAM tool gives us the values of the A matrix (3.3), x vector (3.1) and z vector (3.2), followed by the approach (3.4) and solving the system of equations using the MNA method.

$$x = \begin{bmatrix} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & v_{11} & v_{12} & v_{13} & v_{14} & v_{15} & I_{\mathrm{V1a}} & I_{\mathrm{V1b}} & I_{\mathrm{V1c}} & I_{\mathrm{V2a}} & I_{\mathrm{V2b}} & I_{\mathrm{V2c}} \end{bmatrix}^T \quad (3.1)$$

$$z = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathrm{V1a} & \mathrm{V1b} & \mathrm{V1c} & \mathrm{V2a} & \mathrm{V2b} & \mathrm{V2c} \end{bmatrix}^T \quad (3.2)$$

$$\begin{pmatrix} I_{\mathrm{V1a}} + \frac{v_1}{\mathrm{R1a}} - \frac{v_4}{\mathrm{R1a}} \\ I_{\mathrm{V1b}} + \frac{v_2}{\mathrm{R1b}} - \frac{v_5}{\mathrm{R1b}} \\ I_{\mathrm{V1c}} + \frac{v_3}{\mathrm{R1c}} - \frac{v_6}{\mathrm{R1c}} \\ v_4\left(\frac{1}{\mathrm{R1a}} + \frac{1}{\mathrm{L1a}\,s}\right) - \frac{v_1}{\mathrm{R1a}} - \frac{v_7}{\mathrm{L1a}\,s} \\ v_5\left(\frac{1}{\mathrm{R1b}} + \frac{1}{\mathrm{L1b}\,s}\right) - \frac{v_2}{\mathrm{R1b}} - \frac{v_8}{\mathrm{L1b}\,s} \\ v_6\left(\frac{1}{\mathrm{R1c}} + \frac{1}{\mathrm{L1c}\,s}\right) - \frac{v_3}{\mathrm{R1c}} - \frac{v_9}{\mathrm{L1c}\,s} \\ v_7\left(\frac{1}{\mathrm{RLa}} + \frac{1}{\mathrm{L1a}\,s} + \frac{1}{\mathrm{L2a}\,s}\right) - \frac{v_4}{\mathrm{L1a}\,s} - \frac{v_{10}}{\mathrm{L2a}\,s} \\ v_8\left(\frac{1}{\mathrm{RLb}} + \frac{1}{\mathrm{L1b}\,s} + \frac{1}{\mathrm{L2b}\,s}\right) - \frac{v_5}{\mathrm{L1b}\,s} - \frac{v_{11}}{\mathrm{L2b}\,s} \\ v_9\left(\frac{1}{\mathrm{RLc}} + \frac{1}{\mathrm{L1c}\,s} + \frac{1}{\mathrm{L2c}\,s}\right) - \frac{v_6}{\mathrm{L1c}\,s} - \frac{v_{12}}{\mathrm{L2c}\,s} \\ v_{10}\left(\frac{1}{\mathrm{R2a}} + \frac{1}{\mathrm{L2a}\,s}\right) - \frac{v_{13}}{\mathrm{R2a}} - \frac{v_7}{\mathrm{L2a}\,s} \\ v_{11}\left(\frac{1}{\mathrm{R2b}} + \frac{1}{\mathrm{L2b}\,s}\right) - \frac{v_{14}}{\mathrm{R2b}} - \frac{v_8}{\mathrm{L2b}\,s} \\ v_{12}\left(\frac{1}{\mathrm{R2c}} + \frac{1}{\mathrm{L2c}\,s}\right) - \frac{v_{15}}{\mathrm{R2c}} - \frac{v_9}{\mathrm{L2c}\,s} \\ I_{\mathrm{V2a}} - \frac{v_{10}}{\mathrm{R2a}} + \frac{v_{13}}{\mathrm{R2a}} \\ I_{\mathrm{V2b}} - \frac{v_{11}}{\mathrm{R2b}} + \frac{v_{14}}{\mathrm{R2b}} \\ I_{\mathrm{V2c}} - \frac{v_{12}}{\mathrm{R2c}} + \frac{v_{15}}{\mathrm{R2c}} \\ v_1 \\ v_2 \\ v_3 \\ v_{13} \\ v_{14} \\ v_{15} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \mathrm{V1a} \\ \mathrm{V1b} \\ \mathrm{V1c} \\ \mathrm{V2a} \\ \mathrm{V2b} \\ \mathrm{V2c} \end{pmatrix} \tag{3.4}$$

The expression in (3.4) show us th equations system that represent components interconnection in the proposed MG.

$$A=\left[\begin{array}{ccccccccccccccccccccc}
\frac{1}{\mathrm{R1a}} & 0 & 0 & -\frac{1}{\mathrm{R1a}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
0 & \frac{1}{\mathrm{R1b}} & 0 & 0 & -\frac{1}{\mathrm{R1b}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\
0 & 0 & \frac{1}{\mathrm{R1c}} & 0 & 0 & -\frac{1}{\mathrm{R1c}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\
-\frac{1}{\mathrm{R1a}} & 0 & 0 & \frac{1}{\mathrm{R1a}}+\frac{1}{\mathrm{L1a}\,s} & 0 & 0 & -\frac{1}{\mathrm{L1a}\,s} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -\frac{1}{\mathrm{R1b}} & 0 & 0 & \frac{1}{\mathrm{R1b}}+\frac{1}{\mathrm{L1b}\,s} & 0 & 0 & -\frac{1}{\mathrm{L1b}\,s} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & -\frac{1}{\mathrm{R1c}} & 0 & 0 & \frac{1}{\mathrm{R1c}}+\frac{1}{\mathrm{L1c}\,s} & 0 & 0 & -\frac{1}{\mathrm{L1c}\,s} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -\frac{1}{\mathrm{L1a}\,s} & 0 & 0 & \frac{1}{\mathrm{RLa}}+\frac{1}{\mathrm{L1a}\,s}+\frac{1}{\mathrm{L2a}\,s} & 0 & 0 & -\frac{1}{\mathrm{L2a}\,s} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & -\frac{1}{\mathrm{L1b}\,s} & 0 & 0 & \frac{1}{\mathrm{RLb}}+\frac{1}{\mathrm{L1b}\,s}+\frac{1}{\mathrm{L2b}\,s} & 0 & 0 & -\frac{1}{\mathrm{L2b}\,s} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -\frac{1}{\mathrm{L1c}\,s} & 0 & 0 & \frac{1}{\mathrm{RLc}}+\frac{1}{\mathrm{L1c}\,s}+\frac{1}{\mathrm{L2c}\,s} & 0 & 0 & -\frac{1}{\mathrm{L2c}\,s} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\mathrm{L2a}\,s} & 0 & 0 & \frac{1}{\mathrm{R2a}}+\frac{1}{\mathrm{L2a}\,s} & 0 & 0 & -\frac{1}{\mathrm{R2a}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\mathrm{L2b}\,s} & 0 & 0 & \frac{1}{\mathrm{R2b}}+\frac{1}{\mathrm{L2b}\,s} & 0 & 0 & -\frac{1}{\mathrm{R2b}} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\mathrm{L2c}\,s} & 0 & 0 & \frac{1}{\mathrm{R2c}}+\frac{1}{\mathrm{L2c}\,s} & 0 & 0 & -\frac{1}{\mathrm{R2c}} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\mathrm{R2a}} & 0 & 0 & \frac{1}{\mathrm{R2a}} & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\mathrm{R2b}} & 0 & 0 & \frac{1}{\mathrm{R2b}} & 0 & 0 & 0 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\mathrm{R2c}} & 0 & 0 & \frac{1}{\mathrm{R2c}} & 0 & 0 & 0 & 0 & 0 & 1\\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right] \tag{3.3}$$

3.3 Computer analysis of State Space model

Once SCAM obtains the solution, my interest turns to the equations that relate the currents as a function of the voltages expressed in (2.43) that was explained in section 2.3, and I obtain the matrix of transfer functions that relate each output to each input of the system using Matlab functions. To obtain the transfer function (3.5) it is necessary to know that the inputs of the electrical circuit correspond to the voltage in each source or inverter. In this work, two three-phase sources V_{1a}, V_{1b}, V_{1c} and V_{2a}, V_{2b}, V_{2c} are presented, and as outputs of the system we have currents I_{V1a}, I_{V1b}, I_{V1c} and I_{V2a}, I_{V2b}, I_{V2c} respectively.

$$G_{ij}(s) = \frac{Y(s)}{U(s)} = \frac{I_{\mathrm{Vij}}(s)}{V_{ij}(s)}, \quad i = 1,2 \quad j = a,b,c \tag{3.5}$$

With the values obtained from the SCAM tool and stored in the workspace, we extract the output variables I_{V1a} (3.6), I_{V1b} (3.8), I_{V1c} (3.10) and I_{V2a} (3.12), I_{V2b} (3.14), I_{V2c} (3.16), each based on inputs V_{1a}, V_{1b}, V_{1c} and V_{2a}, V_{2b}, V_{2c}.

$$I_{\mathrm{V1a}}(s) = \frac{\mathrm{R2a\,V1a} + \mathrm{RLa\,V1a} - \mathrm{RLa\,V2a} + \mathrm{L2a\,V1a}\,s}{\mathrm{R1a\,R2a} + \mathrm{R1a\,RLa} + \mathrm{R2a\,RLa} + \mathrm{L1a\,R2a}\,s + \mathrm{L2a\,R1a}\,s + \mathrm{L1a\,RLa}\,s + \mathrm{L2a\,RLa}\,s + \mathrm{L1a\,L2a}\,s^2} \tag{3.6}$$

$$\begin{aligned} \Rightarrow I_{\mathrm{V1a}}(s) = &\left(\frac{\mathrm{R2a} + \mathrm{RLa} + \mathrm{L2a}\,s}{\mathrm{R1a\,R2a} + \mathrm{R1a\,RLa} + \mathrm{R2a\,RLa} + \mathrm{L1a\,R2a}\,s + \mathrm{L2a\,R1a}\,s + \mathrm{L1a\,RLa}\,s + \mathrm{L2a\,RLa}\,s + \mathrm{L1a\,L2a}\,s^2}\right)\mathrm{V1a} \\ &+\left(-\frac{\mathrm{RLa}}{\mathrm{R1a\,R2a} + \mathrm{R1a\,RLa} + \mathrm{R2a\,RLa} + \mathrm{L1a\,R2a}\,s + \mathrm{L2a\,R1a}\,s + \mathrm{L1a\,RLa}\,s + \mathrm{L2a\,RLa}\,s + \mathrm{L1a\,L2a}\,s^2}\right)\mathrm{V2a} \end{aligned} \tag{3.7}$$

In order to more easily recognize the coefficients of each input variable V_{1a}, V_{1b}, V_{1c} and V_{2a}, V_{2b}, V_{2c}, we make use of the *coeffs* command, obtaining better equations distributed in (3.7), (3.9), (3.11) and (3.13), (3.15), (3.17).

$$I_{\mathrm{V1b}}(s) = \frac{\mathrm{R2b\,V1b} + \mathrm{RLb\,V1b} - \mathrm{RLb\,V2b} + \mathrm{L2b\,V1b}\,s}{\mathrm{R1b\,R2b} + \mathrm{R1b\,RLb} + \mathrm{R2b\,RLb} + \mathrm{L1b\,R2b}\,s + \mathrm{L2b\,R1b}\,s + \mathrm{L1b\,RLb}\,s + \mathrm{L2b\,RLb}\,s + \mathrm{L1b\,L2b}\,s^2} \tag{3.8}$$

$$\begin{aligned} \Rightarrow I_{\mathrm{V1b}}(s) = &\left(\frac{\mathrm{R2b} + \mathrm{RLb} + \mathrm{L2b}\,s}{\mathrm{R1b\,R2b} + \mathrm{R1b\,RLb} + \mathrm{R2b\,RLb} + \mathrm{L1b\,R2b}\,s + \mathrm{L2b\,R1b}\,s + \mathrm{L1b\,RLb}\,s + \mathrm{L2b\,RLb}\,s + \mathrm{L1b\,L2b}\,s^2}\right)\mathrm{V1b} \\ &+\left(-\frac{\mathrm{RLb}}{\mathrm{R1b\,R2b} + \mathrm{R1b\,RLb} + \mathrm{R2b\,RLb} + \mathrm{L1b\,R2b}\,s + \mathrm{L2b\,R1b}\,s + \mathrm{L1b\,RLb}\,s + \mathrm{L2b\,RLb}\,s + \mathrm{L1b\,L2b}\,s^2}\right)\mathrm{V2b} \end{aligned} \tag{3.9}$$

$$I_{\mathrm{V1c}}(s) = \frac{\mathrm{R2c\,V1c} + \mathrm{RLc\,V1c} - \mathrm{RLc\,V2c} + \mathrm{L2c\,V1c}\,s}{\mathrm{R1c\,R2c} + \mathrm{R1c\,RLc} + \mathrm{R2c\,RLc} + \mathrm{L1c\,R2c}\,s + \mathrm{L2c\,R1c}\,s + \mathrm{L1c\,RLc}\,s + \mathrm{L2c\,RLc}\,s + \mathrm{L1c\,L2c}\,s^2} \tag{3.10}$$

$$\Rightarrow I_{\mathrm{V1c}}(s) = \left(\frac{\mathrm{R2c} + \mathrm{RLc} + \mathrm{L2c}\, s}{\mathrm{R1c}\,\mathrm{R2c} + \mathrm{R1c}\,\mathrm{RLc} + \mathrm{R2c}\,\mathrm{RLc} + \mathrm{L1c}\,\mathrm{R2c}\, s + \mathrm{L2c}\,\mathrm{R1c}\, s + \mathrm{L1c}\,\mathrm{RLc}\, s + \mathrm{L2c}\,\mathrm{RLc}\, s + \mathrm{L1c}\,\mathrm{L2c}\, s^2}\right) \mathrm{V1c} + \left(-\frac{\mathrm{RLc}}{\mathrm{R1c}\,\mathrm{R2c} + \mathrm{R1c}\,\mathrm{RLc} + \mathrm{R2c}\,\mathrm{RLc} + \mathrm{L1c}\,\mathrm{R2c}\, s + \mathrm{L2c}\,\mathrm{R1c}\, s + \mathrm{L1c}\,\mathrm{RLc}\, s + \mathrm{L2c}\,\mathrm{RLc}\, s + \mathrm{L1c}\,\mathrm{L2c}\, s^2}\right) \mathrm{V2c} \tag{3.11}$$

$$I_{\mathrm{V2a}}(s) = \frac{\mathrm{R1a}\,\mathrm{V2a} - \mathrm{RLa}\,\mathrm{V1a} + \mathrm{RLa}\,\mathrm{V2a} + \mathrm{L1a}\,\mathrm{V2a}\, s}{\mathrm{R1a}\,\mathrm{R2a} + \mathrm{R1a}\,\mathrm{RLa} + \mathrm{R2a}\,\mathrm{RLa} + \mathrm{L1a}\,\mathrm{R2a}\, s + \mathrm{L2a}\,\mathrm{R1a}\, s + \mathrm{L1a}\,\mathrm{RLa}\, s + \mathrm{L2a}\,\mathrm{RLa}\, s + \mathrm{L1a}\,\mathrm{L2a}\, s^2} \tag{3.12}$$

$$\Rightarrow I_{\mathrm{V2a}}(s) = \left(-\frac{\mathrm{RLa}}{\mathrm{R1a}\,\mathrm{R2a} + \mathrm{R1a}\,\mathrm{RLa} + \mathrm{R2a}\,\mathrm{RLa} + \mathrm{L1a}\,\mathrm{R2a}\, s + \mathrm{L2a}\,\mathrm{R1a}\, s + \mathrm{L1a}\,\mathrm{RLa}\, s + \mathrm{L2a}\,\mathrm{RLa}\, s + \mathrm{L1a}\,\mathrm{L2a}\, s^2}\right) \mathrm{V1a} + \left(\frac{\mathrm{R1a} + \mathrm{RLa} + \mathrm{L1a}\, s}{\mathrm{R1a}\,\mathrm{R2a} + \mathrm{R1a}\,\mathrm{RLa} + \mathrm{R2a}\,\mathrm{RLa} + \mathrm{L1a}\,\mathrm{R2a}\, s + \mathrm{L2a}\,\mathrm{R1a}\, s + \mathrm{L1a}\,\mathrm{RLa}\, s + \mathrm{L2a}\,\mathrm{RLa}\, s + \mathrm{L1a}\,\mathrm{L2a}\, s^2}\right) \mathrm{V2a} \tag{3.13}$$

$$I_{\mathrm{V2b}}(s) = \frac{\mathrm{R1b}\,\mathrm{V2b} - \mathrm{RLb}\,\mathrm{V1b} + \mathrm{RLb}\,\mathrm{V2b} + \mathrm{L1b}\,\mathrm{V2b}\, s}{\mathrm{R1b}\,\mathrm{R2b} + \mathrm{R1b}\,\mathrm{RLb} + \mathrm{R2b}\,\mathrm{RLb} + \mathrm{L1b}\,\mathrm{R2b}\, s + \mathrm{L2b}\,\mathrm{R1b}\, s + \mathrm{L1b}\,\mathrm{RLb}\, s + \mathrm{L2b}\,\mathrm{RLb}\, s + \mathrm{L1b}\,\mathrm{L2b}\, s^2} \tag{3.14}$$

$$\Rightarrow I_{\mathrm{V2b}}(s) = \left(-\frac{\mathrm{RLb}}{\mathrm{R1b}\,\mathrm{R2b} + \mathrm{R1b}\,\mathrm{RLb} + \mathrm{R2b}\,\mathrm{RLb} + \mathrm{L1b}\,\mathrm{R2b}\, s + \mathrm{L2b}\,\mathrm{R1b}\, s + \mathrm{L1b}\,\mathrm{RLb}\, s + \mathrm{L2b}\,\mathrm{RLb}\, s + \mathrm{L1b}\,\mathrm{L2b}\, s^2}\right) \mathrm{V1b} + \left(\frac{\mathrm{R1b} + \mathrm{RLb} + \mathrm{L1b}\, s}{\mathrm{R1b}\,\mathrm{R2b} + \mathrm{R1b}\,\mathrm{RLb} + \mathrm{R2b}\,\mathrm{RLb} + \mathrm{L1b}\,\mathrm{R2b}\, s + \mathrm{L2b}\,\mathrm{R1b}\, s + \mathrm{L1b}\,\mathrm{RLb}\, s + \mathrm{L2b}\,\mathrm{RLb}\, s + \mathrm{L1b}\,\mathrm{L2b}\, s^2}\right) \mathrm{V2b} \tag{3.15}$$

$$I_{\mathrm{V2c}}(s) = \frac{\mathrm{R1c}\,\mathrm{V2c} - \mathrm{RLc}\,\mathrm{V1c} + \mathrm{RLc}\,\mathrm{V2c} + \mathrm{L1c}\,\mathrm{V2c}\, s}{\mathrm{R1c}\,\mathrm{R2c} + \mathrm{R1c}\,\mathrm{RLc} + \mathrm{R2c}\,\mathrm{RLc} + \mathrm{L1c}\,\mathrm{R2c}\, s + \mathrm{L2c}\,\mathrm{R1c}\, s + \mathrm{L1c}\,\mathrm{RLc}\, s + \mathrm{L2c}\,\mathrm{RLc}\, s + \mathrm{L1c}\,\mathrm{L2c}\, s^2} \tag{3.16}$$

$$\Rightarrow I_{\mathrm{V2c}}(s) = \left(-\frac{\mathrm{RLc}}{\mathrm{R1c}\,\mathrm{R2c} + \mathrm{R1c}\,\mathrm{RLc} + \mathrm{R2c}\,\mathrm{RLc} + \mathrm{L1c}\,\mathrm{R2c}\, s + \mathrm{L2c}\,\mathrm{R1c}\, s + \mathrm{L1c}\,\mathrm{RLc}\, s + \mathrm{L2c}\,\mathrm{RLc}\, s + \mathrm{L1c}\,\mathrm{L2c}\, s^2}\right) \mathrm{V1c} + \left(\frac{\mathrm{R1c} + \mathrm{RLc} + \mathrm{L1c}\, s}{\mathrm{R1c}\,\mathrm{R2c} + \mathrm{R1c}\,\mathrm{RLc} + \mathrm{R2c}\,\mathrm{RLc} + \mathrm{L1c}\,\mathrm{R2c}\, s + \mathrm{L2c}\,\mathrm{R1c}\, s + \mathrm{L1c}\,\mathrm{RLc}\, s + \mathrm{L2c}\,\mathrm{RLc}\, s + \mathrm{L1c}\,\mathrm{L2c}\, s^2}\right) \mathrm{V2c} \tag{3.17}$$

In this work we have a system with six inputs and six outputs, in which each output depends on the inputs, therefore, we will have a 6×6 matrix with thirty six transfer functions corresponding to (3.18).

$$G(s) = \begin{bmatrix} G_{1,1}(s) & G_{1,2}(s) & G_{1,3}(s) & G_{1,4}(s) & G_{1,5}(s) & G_{1,6}(s) \\ G_{2,1}(s) & G_{2,2}(s) & G_{2,3}(s) & G_{2,4}(s) & G_{2,5}(s) & G_{2,6}(s) \\ G_{3,1}(s) & G_{3,2}(s) & G_{3,3}(s) & G_{3,4}(s) & G_{3,5}(s) & G_{3,6}(s) \\ G_{4,1}(s) & G_{4,2}(s) & G_{4,3}(s) & G_{4,4}(s) & G_{4,5}(s) & G_{4,6}(s) \\ G_{5,1}(s) & G_{5,2}(s) & G_{5,3}(s) & G_{5,4}(s) & G_{5,5}(s) & G_{5,6}(s) \\ G_{6,1}(s) & G_{6,2}(s) & G_{6,3}(s) & G_{6,4}(s) & G_{6,5}(s) & G_{6,6}(s) \end{bmatrix} \tag{3.18}$$

where:

$$
\begin{aligned}
G_{1,1}(s) =& \frac{I_{\mathrm{V1a}}(s)}{V_{1a}(s)} = \left(\frac{\mathrm{R2a} + \mathrm{RLa} + \mathrm{L2a}\,s}{\mathrm{R1a}\,\mathrm{R2a} + \mathrm{R1a}\,\mathrm{RLa} + \mathrm{R2a}\,\mathrm{RLa} + \mathrm{L1a}\,\mathrm{R2a}\,s + \mathrm{L2a}\,\mathrm{R1a}\,s + \mathrm{L1a}\,\mathrm{RLa}\,s + \mathrm{L2a}\,\mathrm{RLa}\,s + \mathrm{L1a}\,\mathrm{L2a}\,s^2} \right)\\
G_{1,2}(s) =& \frac{I_{\mathrm{V1a}}(s)}{V_{1b}(s)} = 0\\
G_{1,3}(s) =& \frac{I_{\mathrm{V1a}}(s)}{V_{1c}(s)} = 0\\
G_{1,4}(s) =& \frac{I_{\mathrm{V1a}}(s)}{V_{2a}(s)} = \left(-\frac{\mathrm{RLa}}{\mathrm{R1a}\,\mathrm{R2a} + \mathrm{R1a}\,\mathrm{RLa} + \mathrm{R2a}\,\mathrm{RLa} + \mathrm{L1a}\,\mathrm{R2a}\,s + \mathrm{L2a}\,\mathrm{R1a}\,s + \mathrm{L1a}\,\mathrm{RLa}\,s + \mathrm{L2a}\,\mathrm{RLa}\,s + \mathrm{L1a}\,\mathrm{L2a}\,s^2} \right)\\
G_{1,5}(s) =& \frac{I_{\mathrm{V1a}}(s)}{V_{2b}(s)} = 0\\
G_{1,6}(s) =& \frac{I_{\mathrm{V1a}}(s)}{V_{2c}(s)} = 0
\end{aligned}
\tag{3.19}
$$

$$
\begin{aligned}
G_{2,1}(s) =& \frac{I_{\mathrm{V1b}}(s)}{V_{1a}(s)} = 0\\
G_{2,2}(s) =& \frac{I_{\mathrm{V1b}}(s)}{V_{1b}(s)} = \left(\frac{\mathrm{R2b} + \mathrm{RLb} + \mathrm{L2b}\,s}{\mathrm{R1b}\,\mathrm{R2b} + \mathrm{R1b}\,\mathrm{RLb} + \mathrm{R2b}\,\mathrm{RLb} + \mathrm{L1b}\,\mathrm{R2b}\,s + \mathrm{L2b}\,\mathrm{R1b}\,s + \mathrm{L1b}\,\mathrm{RLb}\,s + \mathrm{L2b}\,\mathrm{RLb}\,s + \mathrm{L1b}\,\mathrm{L2b}\,s^2} \right)\\
G_{2,3}(s) =& \frac{I_{\mathrm{V1b}}(s)}{V_{1c}(s)} = 0\\
G_{2,4}(s) =& \frac{I_{\mathrm{V1b}}(s)}{V_{2a}(s)} = 0\\
G_{2,5}(s) =& \frac{I_{\mathrm{V1b}}(s)}{V_{2b}(s)} = \left(-\frac{\mathrm{RLb}}{\mathrm{R1b}\,\mathrm{R2b} + \mathrm{R1b}\,\mathrm{RLb} + \mathrm{R2b}\,\mathrm{RLb} + \mathrm{L1b}\,\mathrm{R2b}\,s + \mathrm{L2b}\,\mathrm{R1b}\,s + \mathrm{L1b}\,\mathrm{RLb}\,s + \mathrm{L2b}\,\mathrm{RLb}\,s + \mathrm{L1b}\,\mathrm{L2b}\,s^2} \right)\\
G_{2,6}(s) =& \frac{I_{\mathrm{V1b}}(s)}{V_{2c}(s)} = 0
\end{aligned}
\tag{3.20}
$$

$$
\begin{aligned}
G_{3,1}(s) =& \frac{I_{\mathrm{V1c}}(s)}{V_{1a}(s)} = 0\\
G_{3,2}(s) =& \frac{I_{\mathrm{V1c}}(s)}{V_{1b}(s)} = 0\\
G_{3,3}(s) =& \frac{I_{\mathrm{V1c}}(s)}{V_{1c}(s)} = \left(\frac{\mathrm{R2c} + \mathrm{RLc} + \mathrm{L2c}\,s}{\mathrm{R1c}\,\mathrm{R2c} + \mathrm{R1c}\,\mathrm{RLc} + \mathrm{R2c}\,\mathrm{RLc} + \mathrm{L1c}\,\mathrm{R2c}\,s + \mathrm{L2c}\,\mathrm{R1c}\,s + \mathrm{L1c}\,\mathrm{RLc}\,s + \mathrm{L2c}\,\mathrm{RLc}\,s + \mathrm{L1c}\,\mathrm{L2c}\,s^2} \right)\\
G_{3,4}(s) =& \frac{I_{\mathrm{V1c}}(s)}{V_{2a}(s)} = 0\\
G_{3,5}(s) =& \frac{I_{\mathrm{V1c}}(s)}{V_{2b}(s)} = 0\\
G_{3,6}(s) =& \frac{I_{\mathrm{V1c}}(s)}{V_{2c}(s)} = \left(-\frac{\mathrm{RLc}}{\mathrm{R1c}\,\mathrm{R2c} + \mathrm{R1c}\,\mathrm{RLc} + \mathrm{R2c}\,\mathrm{RLc} + \mathrm{L1c}\,\mathrm{R2c}\,s + \mathrm{L2c}\,\mathrm{R1c}\,s + \mathrm{L1c}\,\mathrm{RLc}\,s + \mathrm{L2c}\,\mathrm{RLc}\,s + \mathrm{L1c}\,\mathrm{L2c}\,s^2} \right)
\end{aligned}
\tag{3.21}
$$

$$
\begin{aligned}
G_{4,1}(s) &= \frac{I_{\mathrm{V2a}}(s)}{V_{1a}(s)} = \left(-\frac{\mathrm{RLa}}{\mathrm{R1a\,R2a} + \mathrm{R1a\,RLa} + \mathrm{R2a\,RLa} + \mathrm{L1a\,R2a}\,s + \mathrm{L2a\,R1a}\,s + \mathrm{L1a\,RLa}\,s + \mathrm{L2a\,RLa}\,s + \mathrm{L1a\,L2a}\,s^2}\right)\\
G_{4,2}(s) &= \frac{I_{\mathrm{V2a}}(s)}{V_{1b}(s)} = 0\\
G_{4,3}(s) &= \frac{I_{\mathrm{V2a}}(s)}{V_{1c}(s)} = 0\\
G_{4,4}(s) &= \frac{I_{\mathrm{V2a}}(s)}{V_{2a}(s)} = \left(\frac{\mathrm{R1a} + \mathrm{RLa} + \mathrm{L1a}\,s}{\mathrm{R1a\,R2a} + \mathrm{R1a\,RLa} + \mathrm{R2a\,RLa} + \mathrm{L1a\,R2a}\,s + \mathrm{L2a\,R1a}\,s + \mathrm{L1a\,RLa}\,s + \mathrm{L2a\,RLa}\,s + \mathrm{L1a\,L2a}\,s^2}\right)\\
G_{4,5}(s) &= \frac{I_{\mathrm{V2a}}(s)}{V_{2b}(s)} = 0\\
G_{4,6}(s) &= \frac{I_{\mathrm{V2a}}(s)}{V_{2c}(s)} = 0
\end{aligned}
\tag{3.22}
$$

$$
\begin{aligned}
G_{5,1}(s) &= \frac{I_{\mathrm{V2b}}(s)}{V_{1a}(s)} = 0\\
G_{5,2}(s) &= \frac{I_{\mathrm{V2b}}(s)}{V_{1b}(s)} = \left(-\frac{\mathrm{RLb}}{\mathrm{R1b\,R2b} + \mathrm{R1b\,RLb} + \mathrm{R2b\,RLb} + \mathrm{L1b\,R2b}\,s + \mathrm{L2b\,R1b}\,s + \mathrm{L1b\,RLb}\,s + \mathrm{L2b\,RLb}\,s + \mathrm{L1b\,L2b}\,s^2}\right)\\
G_{5,3}(s) &= \frac{I_{\mathrm{V2b}}(s)}{V_{1c}(s)} = 0\\
G_{5,4}(s) &= \frac{I_{\mathrm{V2b}}(s)}{V_{2a}(s)} = 0\\
G_{5,5}(s) &= \frac{I_{\mathrm{V2b}}(s)}{V_{2b}(s)} = \left(\frac{\mathrm{R1b} + \mathrm{RLb} + \mathrm{L1b}\,s}{\mathrm{R1b\,R2b} + \mathrm{R1b\,RLb} + \mathrm{R2b\,RLb} + \mathrm{L1b\,R2b}\,s + \mathrm{L2b\,R1b}\,s + \mathrm{L1b\,RLb}\,s + \mathrm{L2b\,RLb}\,s + \mathrm{L1b\,L2b}\,s^2}\right)\\
G_{5,6}(s) &= \frac{I_{\mathrm{V2b}}(s)}{V_{2c}(s)} = 0
\end{aligned}
\tag{3.23}
$$

$$
\begin{aligned}
G_{6,1}(s) &= \frac{I_{\mathrm{V2c}}(s)}{V_{1a}(s)} = 0\\
G_{6,2}(s) &= \frac{I_{\mathrm{V2c}}(s)}{V_{1b}(s)} = 0\\
G_{6,3}(s) &= \frac{I_{\mathrm{V2c}}(s)}{V_{1c}(s)} = \left(-\frac{\mathrm{RLc}}{\mathrm{R1c\,R2c} + \mathrm{R1c\,RLc} + \mathrm{R2c\,RLc} + \mathrm{L1c\,R2c}\,s + \mathrm{L2c\,R1c}\,s + \mathrm{L1c\,RLc}\,s + \mathrm{L2c\,RLc}\,s + \mathrm{L1c\,L2c}\,s^2}\right)\\
G_{6,4}(s) &= \frac{I_{\mathrm{V2c}}(s)}{V_{2a}(s)} = 0\\
G_{6,5}(s) &= \frac{I_{\mathrm{V2c}}(s)}{V_{2b}(s)} = 0\\
G_{6,6}(s) &= \frac{I_{\mathrm{V2c}}(s)}{V_{2c}(s)} = \left(\frac{\mathrm{R1c} + \mathrm{RLc} + \mathrm{L1c}\,s}{\mathrm{R1c\,R2c} + \mathrm{R1c\,RLc} + \mathrm{R2c\,RLc} + \mathrm{L1c\,R2c}\,s + \mathrm{L2c\,R1c}\,s + \mathrm{L1c\,RLc}\,s + \mathrm{L2c\,RLc}\,s + \mathrm{L1c\,L2c}\,s^2}\right)
\end{aligned}
\tag{3.24}
$$

Parameter	**Symbol**	**Value**	**Unit**
Inversor 1 (a, b, c)	V_{1a}, V_{1b}, V_{1c}	155	V_p
Inversor 2(a, b, c)	V_{2a}, V_{2b}, V_{2c}	155	V_p
Resistor 1(a, b, c)	R_{1a}, R_{1b}, R_{1c}	0.5	Ω
Resistor 2(a, b, c)	R_{2a}, R_{2b}, R_{2c}	10	Ω
Resistor Load (a, b, c)	R_{La}, R_{Lb}, R_{Lc}	2	Ω
Inductor 1(a, b, c)	L_{1a}, L_{1b}, L_{1c}	20	mH
Inductor 2(a, b, c)	L_{2a}, L_{2b}, L_{2c}	10	mH

Table 10: MG three-phase model parameters

To obtain the results with numerical values, we use the `subs` command in MATLAB considering the MG parameters shown in table 10, achieving a 6×6 matrix of thirty six transfer functions (3.25).

$$\Rightarrow G(s) = \begin{pmatrix} \frac{\frac{s}{100}+12}{\frac{s^2}{5000}+\frac{53\,s}{200}+26} & 0 & 0 & -\frac{2}{\frac{s^2}{5000}+\frac{53\,s}{200}+26} & 0 & 0 \\ 0 & \frac{\frac{s}{100}+12}{\frac{s^2}{5000}+\frac{53\,s}{200}+26} & 0 & 0 & -\frac{2}{\frac{s^2}{5000}+\frac{53\,s}{200}+26} & 0 \\ 0 & 0 & \frac{\frac{s}{100}+12}{\frac{s^2}{5000}+\frac{53\,s}{200}+26} & 0 & 0 & -\frac{2}{\frac{s^2}{5000}+\frac{53\,s}{200}+26} \\ -\frac{2}{\frac{s^2}{5000}+\frac{53\,s}{200}+26} & 0 & 0 & \frac{\frac{s}{50}+\frac{5}{2}}{\frac{s^2}{5000}+\frac{53\,s}{200}+26} & 0 & 0 \\ 0 & -\frac{2}{\frac{s^2}{5000}+\frac{53\,s}{200}+26} & 0 & 0 & \frac{\frac{s}{50}+\frac{5}{2}}{\frac{s^2}{5000}+\frac{53\,s}{200}+26} & 0 \\ 0 & 0 & -\frac{2}{\frac{s^2}{5000}+\frac{53\,s}{200}+26} & 0 & 0 & \frac{\frac{s}{50}+\frac{5}{2}}{\frac{s^2}{5000}+\frac{53\,s}{200}+26} \end{pmatrix} \tag{3.25}$$

To obtain matrices A, B, C and D in (3.27), from the transfer matrix $G(s)$ of the whole system that groups the six outputs based on the six inputs (3.25), we use the script, shown in (Algorithm 2), to convert it in an object type `tf` (transfer function G_{tf}), and this in turn, transform into an object type `ss` (state space G_{ss}) with the command of the same name (3.26), where we can practically obtain the required matrices to continue with the analysis of this work in a state space model.

$$\mathtt{G_{ss}} = \mathtt{ss}\left(\mathtt{Gtf}\right) \tag{3.26}$$

$$
A = G_{ss}.A = \begin{pmatrix}
-1325 & -507.8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
256 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1325 & -507.8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 256 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1325 & -507.8 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 256 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1325 & -507.8 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 256 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1325 & -507.8 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 256 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1325 & -507.8 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 256 & 0
\end{pmatrix}
$$

$$
B = G_{ss}.B = \begin{pmatrix}
16 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 16 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 16 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 8 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 8 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 8 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

$$
C = G_{ss}.C = \begin{pmatrix}
3.125 & 14.65 & 0 & 0 & 0 & 0 & 0 & -4.88 & 0 & 0 & 0 & 0 \\
0 & 0 & 3.125 & 14.65 & 0 & 0 & 0 & 0 & 0 & -4.88 & 0 & 0 \\
0 & 0 & 0 & 0 & 3.125 & 14.65 & 0 & 0 & 0 & 0 & 0 & -4.88 \\
0 & -2.441 & 0 & 0 & 0 & 0 & 12.5 & 6.10 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -2.441 & 0 & 0 & 0 & 0 & 12.5 & 6.10 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -2.441 & 0 & 0 & 0 & 0 & 12.5 & 6.10
\end{pmatrix}
$$

$$
D = G_{ss}.D = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\tag{3.27}
$$

3.4 Simulation of MG of two three-phase inverters with a resistive load

The simulation of the MG three-phase electrical model has been performed with a Matlab script using Euler (2.75) for the states equation (2.50) and the outputs equation (2.51), where the matrices A, B, C and D in (3.27) obtained in the previous section are used.

A simulation time $\mathtt{t_f} = \mathtt{0.1}$ seconds has been configured, with steps $\mathtt{t_s} = 1\mathrm{e} - \mathtt{6}$, a frequency $\omega = 2\pi\mathtt{f} = 2\pi(\mathtt{60})$. In this work, we assume a phase of the first three phase voltage source $\varphi_1 = 0$ and a range of values for the phase of the second three phase voltage source $\varphi_2 = 0 : 0.05 : 0.5$ in order to compare the behavior of the system when the six sources gradually phase out of each other and observe the voltage, current and power signals graphically.

In (3.28), we use inputs one (3.29), (3.30), (3.31) and inputs two (3.32), (3.33), (3.34) of voltage as an Euler's exponential function:

$$\mathtt{u} = \begin{pmatrix} u_{1a} \\ u_{1b} \\ u_{1c} \\ u_{2a} \\ u_{2b} \\ u_{2c} \end{pmatrix} \tag{3.28}$$

where:

$$\mathtt{u_{1a}(t)} = \mathtt{u_0\, e}^{\mathtt{j}\omega\,\mathtt{t}+\mathtt{j}(\theta_{1a}+\varphi_1)}, \quad \mathtt{u_0} = 155, \quad \theta_{1a} = 0 \tag{3.29}$$

$$\mathtt{u_{1b}(t)} = \mathtt{u_0\, e}^{\mathtt{j}\omega\,\mathtt{t}+\mathtt{j}(\theta_{1b}+\varphi_1)}, \quad \mathtt{u_0} = 155, \quad \theta_{1b} = \theta_{1a} + 2\frac{\pi}{3} \tag{3.30}$$

$$\mathtt{u_{1c}(t)} = \mathtt{u_0\, e}^{\mathtt{j}\omega\,\mathtt{t}+\mathtt{j}(\theta_{1c}+\varphi_1)}, \quad \mathtt{u_0} = 155, \quad \theta_{1c} = \theta_{1b} + 2\frac{\pi}{3} \tag{3.31}$$

$$\mathtt{u_{2a}(t)} = \mathtt{u_0\, e}^{\mathtt{j}\omega\,\mathtt{t}+\mathtt{j}(\theta_{2a}+\varphi_2)}, \quad \mathtt{u_0} = 155, \quad \theta_{2a} = 0 \tag{3.32}$$

$$\mathtt{u_{2b}(t)} = \mathtt{u_0\, e}^{\mathtt{j}\omega\,\mathtt{t}+\mathtt{j}(\theta_{2b}+\varphi_2)}, \quad \mathtt{u_0} = 155, \quad \theta_{2b} = \theta_{2a} + 2\frac{\pi}{3} \tag{3.33}$$

$$\mathtt{u_{2c}(t)} = \mathtt{u_0\, e}^{\mathtt{j}\omega\,\mathtt{t}+\mathtt{j}(\theta_{2c}+\varphi_2)}, \quad \mathtt{u_0} = 155, \quad \theta_{2c} = \theta_{2b} + 2\frac{\pi}{3} \tag{3.34}$$

To perform this simulation, we use Euler in the states equation (2.50) as shown in (3.35):

$$\begin{aligned} \mathtt{x(i+1)} &= \mathtt{x(i)} + \mathtt{t_s}\, \mathtt{x} \\ \Rightarrow \mathtt{x(i+1)} &= \mathtt{x(i)} + \mathtt{t_s}\, (\mathtt{A\, x(i)} + \mathtt{B\, u})\,, \quad \mathtt{i} = 0 : \mathtt{length}\left(\frac{\mathtt{t_f}}{\mathtt{t_s}}\right) \end{aligned} \tag{3.35}$$

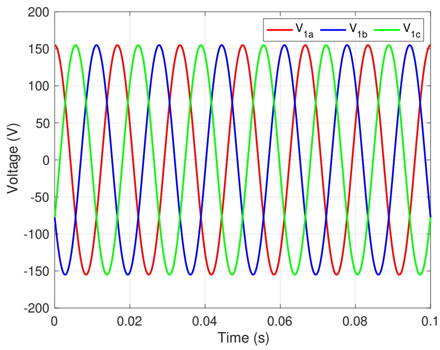
And the output equation (2.51), in this example, is given by (3.36);

$$\mathtt{I} = \mathtt{C\, x} \tag{3.36}$$

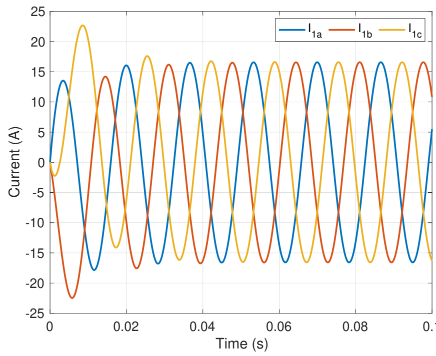
Because the values that are achieved for the voltage and current are stored in a vector format according to the number of inputs and outputs of the system, then the three-phase power of each input can be considered as the sum of the powers of its components a, b, c obtained from the product of these two variables.

Using Matlab script (Section 7.1.2), we obtain the graph corresponding to current, voltage and power of each source. We have carried out the test using two three-phase input sources with equal voltages magnitude, but out of phase.

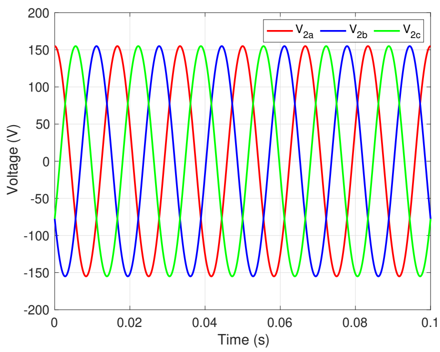
Figure 25 shows the system dynamics with respect to the three-phase one voltage signal (subfigure 25a), the three-phase one current signal (subfigure 25b) and the three-phase two voltage signal (subfigure 25c), the three-phase two current signal (subfigure 25d) when the simulation starts with the phases $\varphi_1 = \varphi_2 = 0$.



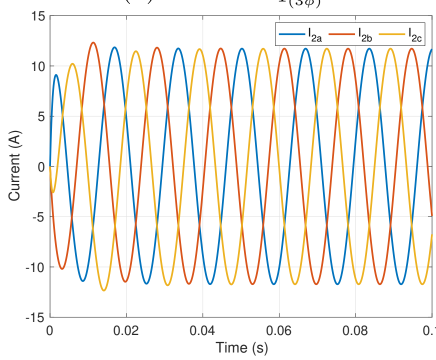
(a) Voltage $V_{1_{(3\phi)}}$



(b) Current $I_{1_{(3\phi)}}$



(c) Voltage $V_{2_{(3\phi)}}$



(d) Current $I_{2_{(3\phi)}}$

Figure 25: System dynamics when two three-phase input sources with equal voltage magnitude and equal phases operate during the simulation time.

The current signal in the subfigure 25b and the subfigure 25d shows a transitory variation for a short period of time at the start of the simulation, until its magnitudes and phases, characteristic of a three-phase signal, are stabilized while the two sources maintain $\varphi_1 = \varphi_2 = 0$.

The active powers of the two three-phase input sources are shown in figure 26, where source one delivers a lower three-phase active power $P_{1_{(3\phi)}}$ than source two, but it produces a significant transient peak until the signal becomes stable, while source two does not suffer a notable variation $P_{2_{(3\phi)}}$ as above until stability is achieved.

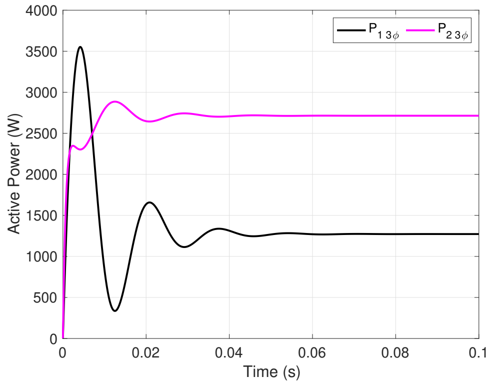
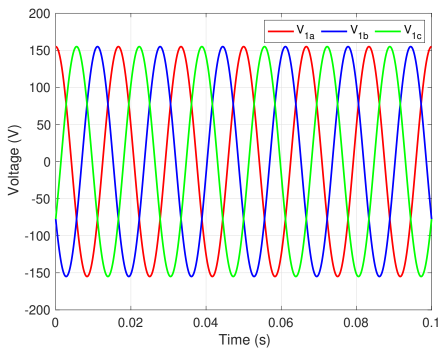


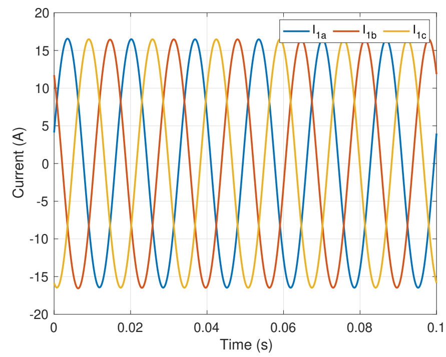
Figure 26: Active Power $P_{(3\phi)}$ of MNA-based MG model

The effect observed in figure 26 of these three-phase power signals is due to the transient variations, in magnitude and phase, of the three-phase currents observed in the subfigure 25b and the subfigure 25d during the start of the simulation.

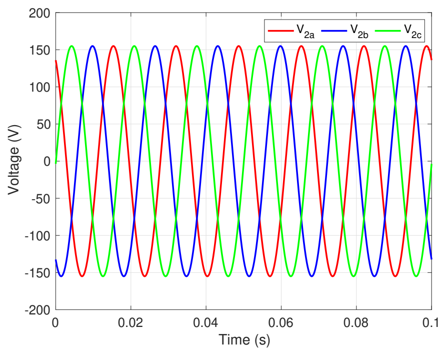
Figure 27 shows the system dynamics with respect to the three-phase one voltage signal (subfigure 27a), the three-phase one current signal (subfigure 27b) and the three-phase two voltage signal (subfigure 27c), the three-phase two current signal (subfigure 27d) when the simulation ends with the phases $\varphi_1 = 0$ and $\varphi_2 = 0.5$. The phase difference between φ_1 and φ_2 of the source voltage signal is programmed in Matlab in a range of 0 to 0.5 with phase increment steps of 0.05 in φ_2 to illustrate the behavior of these signals.



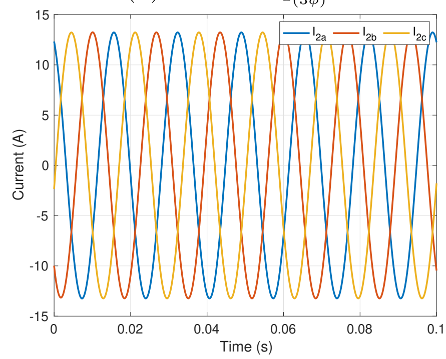
(a) Voltage $V_{1_{(3\phi)}}$



(b) Current $I_{1_{(3\phi)}}$



(c) Voltage $V_{2_{(3\phi)}}$



(d) Current $I_{2_{(3\phi)}}$

Figure 27: System dynamics when two three-phase input sources with equal voltage magnitude and different phases operate during the simulation time.

The current signals in the subfigure 27b and the subfigure 27d don't show a transitory variation in their magnitudes and phases, and remain stable while they are out of phase $\varphi_1 = 0$ and $\varphi_2 = 0.5$ during the simulation time.

The active powers of the two three-phase input sources are shown in figure 28, where the three-phase active power $P_{1_{(3\phi)}}$ of source one is significantly lower than the three-phase active power $P_{2_{(3\phi)}}$ of source two, without a noticeable transient peak occurring in both cases until the signal becomes stable.

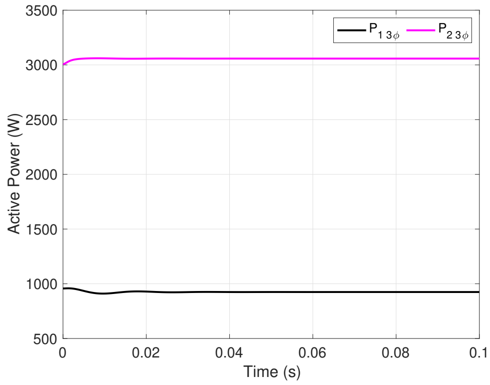


Figure 28: Active Power $P_{(3\phi)}$ of MNA-based MG model

4 Complex-based modeling and analysis of MG of two three-phase inverters with a resistive load

In the previous sections we have learned about the process involved in MG modeling using tools such as MNA through the nodes of the electrical circuit, obtaining a simplified expression of the currents as a function of the voltages, which can be applied computational algorithms that allow to automate the resolution process. As the objective is to obtain the powers delivered by each inverter in the microgrid, it can be done through the following expression

$$P(s) = \mathcal{L}\{i(t)^T v(t)\} = I(s)^T * V(s) = \int_{c-j\omega}^{c+j\omega} I(s)^T V\left(s - j\omega\right) d\omega \quad (4.1)$$

We can already imagine that the convolution integral of product between current and voltage is possible (4.1) but a greater effort is needed to continue with the analysis to obtain the powers, for this reason, it is that from the structure provided by the MNA with SCAM support allows us to find the transfer functions that relate the current and voltage of each inverter and move to the state space, in order to obtain the powers as a product of current and voltage as a function of time in the same algorithmic way to facilitate testing and simulations in Matlab.

All these procedures work properly and allow us to find the powers, but while the microgrid increases in number of inverters, the difficulty to analyze and solve this type of systems increases notably, in addition to the loss of intuition in the interaction of each component of the microgrid, as can be seen in the mathematical expressions (3.3) applying MNA, (3.18) and (3.25) with transfer functions matrix of 6×6 where we only have a microgrid with two three-phase inverters. It is precisely due to the need to find a more compact, simplified, scalable and intuitive alternative that the following complexes-based model is proposed directly from the power equations of each inverter, using the scheme in the figure 29.

4.1 Electrical scheme of three-phase MG

Figure 29 shows the MG scheme being considered in this work, the same one that has been analyzed in figure 12 of section 2.2 using MNA by SCAM, but in three-phase mode.

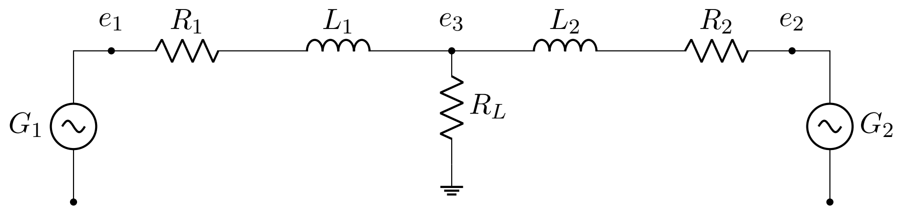


Figure 29: Microgrid scheme

This MG scheme is of the resistive-inductive type (R_1, L_1) and (R_2, L_2) and consists of two three-phase inverters G_1 and G_2 that supply a load pure resistive R_L.

4.2 Complex-based modeling of three-phase MG

Each generator injected complex power is given by

$$S_i(t) = \frac{3}{2}\, e_i(t)\, \bar{i}_i(t), \quad i = 1, 2 \tag{4.2}$$

where $e_i, \bar{i}_i \in \mathbb{C}$ are the complex voltage and the complex conjugate current in $\alpha\beta$-frame, respectively, given by

$$e_i(t) = E_i(t) e^{j\omega_0 t + j\varphi_i(t)} \tag{4.3}$$

$$\bar{i}_i(t) = I_i(t) e^{j\omega_0 t + j\varphi_i'(t)} \tag{4.4}$$

and the constant factor $\frac{3}{2}$ is determinated in active and reactive power expression in the $\alpha\beta$-frame given by

$$P_i(t) = \frac{3}{2}\left(e_{i\alpha}(t) i_{i\alpha}(t) + e_{i\beta}(t) i_{i\beta}(t)\right) \tag{4.5}$$

$$Q_i(t) = \frac{3}{2}\left(-e_{i\alpha}(t) i_{i\beta}(t) + e_{i\beta}(t) i_{i\alpha}(t)\right) \tag{4.6}$$

The complex power can be also written in complex form as

$$S_i = P_i + jQ_i \in \mathbb{C} \tag{4.7}$$

where P_i and Q_i are each generator injected active and reactive power, respectively, and j is the imaginary unit.

In order to study the dynamics of the power, we can differentiate (4.2) to obtain

$$\dot{S}_i(t) = \frac{3}{2}\left(\dot{e}_i(t)\,\bar{i}_i(t) + e_i(t)\,\dot{\bar{i}}_i(t)\right) \tag{4.8}$$

In order to compute (4.8) we need the derivative of the voltage, $\dot{e}_i(t)$, and the derivative of the transpose of the current $\dot{\bar{i}}_i(t)$. The first one is obtained by differentiating (4.3) as follows

$$\begin{aligned}\dot{e}_i(t) &= \dot{E}_i(t)e^{j\omega_0 t + j\varphi_i(t)} + E_i(t)j(\omega_0 + \dot{\varphi}_i(t))e^{j\omega_0 t + j\varphi_i(t)} \\ &= \left(\dot{E}_i(t) + E_i(t)j(\omega_0 + \dot{\varphi}_i(t))\right)e^{j\omega_0 t + j\varphi_i(t)} \\ &= \left(\frac{\dot{E}_i(t)}{E_i(t)} + j(\omega_0 + \dot{\varphi}_i(t))\right)e_i(t)\end{aligned} \tag{4.9}$$

The second one, $\dot{\bar{i}}_i(t)$, is obtained analyzing the considered circuit (remember Figure 29, placing the focus on inverter G_1). Having that

$$L_1\dot{i}_1(t) = e_1(t) - e_3(t) - R_1 i_1(t) \quad \text{and} \quad e_3(t) = R_L(i_1(t) + i_2(t))$$

we obtain that

$$\dot{i}_1(t) = \frac{1}{L_1}e_1(t) - \frac{(R_L + R_1)}{L_1}i_1(t) - \frac{R_L}{L_1}i_2(t) \tag{4.10}$$

The transpose of (4.10) is

$$\dot{\bar{i}}_1(t) = \frac{1}{L_1}\bar{e}_1(t) - \frac{(R_L + R_1)}{L_1}\bar{i}_1(t) - \frac{R_L}{L_1}\bar{i}_2(t) \tag{4.11}$$

By substituing (4.9) and (4.11) into (4.8) for the case of G_1 we obtain

$$\begin{aligned}
\dot{S}_1(t) &= \frac{3}{2}\left(\dot{e}_1(t)\,\bar{i}_1(t) + e_1(t)\,\dot{\bar{i}}_1(t)\right)\\
&= \frac{3}{2}\left(\left(\left(\frac{\dot{E}_1(t)}{E_1(t)} + j(\omega_0 + \dot{\varphi}_1(t))\right)e_1(t)\right)\bar{i}_1(t) + e_1(t)\,\dot{\bar{i}}_1(t)\right)\\
&= \left(\frac{\dot{E}_1(t)}{E_1(t)} + j(\omega_0 + \dot{\varphi}_1(t))\right)S_1(t) + \frac{3}{2}\,e_1(t)\,\dot{\bar{i}}_1(t)\\
&= \left(\frac{\dot{E}_1(t)}{E_1(t)} + j(\omega_0 + \dot{\varphi}_1(t))\right)S_1(t) + \frac{3}{2}\,e_1(t)\left(\frac{1}{L_1}\bar{e}_1(t) - \frac{(R_L+R_1)}{L_1}\bar{i}_1(t) - \frac{R_L}{L_1}\bar{i}_2(t)\right)\\
&= \left(\frac{\dot{E}_1(t)}{E_1(t)} + j(\omega_0 + \dot{\varphi}_1(t)) - \frac{(R_L+R_1)}{L_1}\right)S_1(t) + \frac{3}{2}\,e_1(t)\left(\frac{1}{L_1}\bar{e}_1(t) - \frac{R_L}{L_1}\bar{i}_2(t)\right)\\
&= \left(\frac{\dot{E}_1(t)}{E_1(t)} + j(\omega_0 + \dot{\varphi}_1(t)) - \frac{(R_L+R_1)}{L_1}\right)S_1(t) + \frac{3}{2}\frac{1}{L_1}E_1^2(t) - \frac{3}{2}\frac{R_L}{L_1}e_1(t)\bar{i}_2(t)\\
&= \left(\frac{\dot{E}_1(t)}{E_1(t)} + j(\omega_0 + \dot{\varphi}_1(t)) - \frac{(R_L+R_1)}{L_1}\right)S_1(t) + \frac{3}{2}\frac{1}{L_1}E_1^2(t) - \frac{3}{2}\frac{R_L}{L_1}e_1(t)\bar{i}_2(t)\frac{e_2(t)}{e_2(t)}\\
&= \left(\frac{\dot{E}_1(t)}{E_1(t)} + j(\omega_0 + \dot{\varphi}_1(t)) - \frac{(R_L+R_1)}{L_1}\right)S_1(t) + \frac{3}{2}\frac{E_1^2(t)}{L_1} - \frac{R_L}{L_1}S_2(t)\frac{e_1(t)}{e_2(t)}
\end{aligned} \tag{4.12}$$

Hence, for both inverters, the MG model is

$$\begin{aligned}
\dot{S}_1(t) &= \left(\frac{\dot{E}_1(t)}{E_1(t)} + j(\omega_0 + \dot{\varphi}_1(t)) - \frac{(R_L+R_1)}{L_1}\right)S_1(t) + \frac{3}{2}\frac{E_1^2(t)}{L_1} - \frac{R_L}{L_1}S_2(t)\frac{e_1(t)}{e_2(t)}\\
\dot{S}_2(t) &= \left(\frac{\dot{E}_2(t)}{E_2(t)} + j(\omega_0 + \dot{\varphi}_2(t)) - \frac{(R_L+R_2)}{L_2}\right)S_2(t) + \frac{3}{2}\frac{E_2^2(t)}{L_2} - \frac{R_L}{L_2}S_1(t)\frac{e_2(t)}{e_1(t)}
\end{aligned} \tag{4.13}$$

By defining

$$\alpha = \frac{e_1(t)}{e_2(t)} = \frac{E1}{E2}e^{j(\varphi_1(t)-\varphi_2(t))}$$

the previous model (4.13) becomes

$$\begin{aligned}
\dot{S}_1(t) &= \left(\frac{\dot{E}_1(t)}{E_1(t)} + j(\omega_0 + \dot{\varphi}_1(t)) - \frac{(R_L+R_1)}{L_1}\right)S_1(t) + \frac{3}{2}\frac{E_1^2(t)}{L_1} - \frac{R_L}{L_1}S_2(t)\alpha\\
\dot{S}_2(t) &= \left(\frac{\dot{E}_2(t)}{E_2(t)} + j(\omega_0 + \dot{\varphi}_2(t)) - \frac{(R_L+R_2)}{L_2}\right)S_2(t) + \frac{3}{2}\frac{E_2^2(t)}{L_2} - \frac{R_L}{L_2}S_1(t)\alpha^{-1}
\end{aligned} \tag{4.14}$$

which can be rewritten in state-space form as

$$\begin{pmatrix} \dot{S}_1(t) \\ \dot{S}_2(t) \end{pmatrix} = \begin{pmatrix} \frac{\dot{E}_1(t)}{E_1(t)} + j(\omega_0 + \dot{\varphi}_1(t)) - \frac{(R_L+R_1)}{L_1} & -\frac{R_L}{L_1}\alpha \\ -\frac{R_L}{L_2}\alpha^{-1} & \frac{\dot{E}_2(t)}{E_2(t)} + j(\omega_0 + \dot{\varphi}_2(t)) - \frac{(R_L+R_2)}{L_2} \end{pmatrix} \begin{pmatrix} S_1(t) \\ S_2(t) \end{pmatrix} + \begin{pmatrix} \frac{3}{2}\frac{1}{L_1} & 0 \\ 0 & \frac{3}{2}\frac{1}{L_2} \end{pmatrix} \begin{pmatrix} E_1^2(t) \\ E_2^2(t) \end{pmatrix} \tag{4.15}$$

4.3 Complex-based simulation of three-phase MG

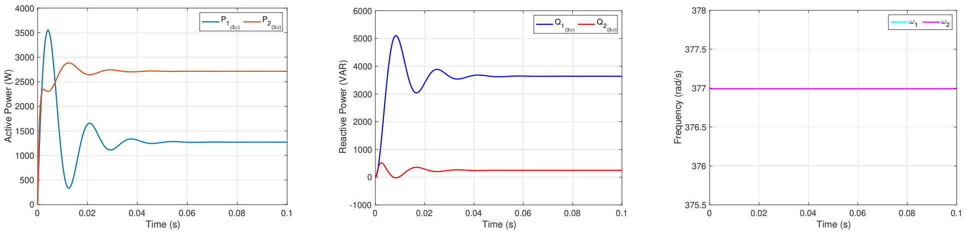
The simulation of the complex-based MG model composed of two three-phase inverters with a resistive load has been carried out with a Matlab script in discrete algorithmically using the model equations system in (4.14) and the expression of voltage in (4.3). To obtain the complex-based MG power model solution, we have considered the parameters shown in the table 11.

Parameter	**Symbol**	**Value**	**Unit**
Inversor 1	G_1	155	V_p
Inversor 2	G_2	155	V_p
Resistor 1	R_1	0.5	Ω
Resistor 2	R_2	10	Ω
Resistor Load	R_L	2	Ω
Inductor 1	L_1	20	mH
Inductor 2	L_2	10	mH

Table 11: Complex-based MG three-phase model parameters

A simulation time $\mathtt{t_f} = \mathtt{0.1}$ seconds has been configured, with steps $\mathtt{t_s} = 1\mathrm{e} - \mathtt{6}$, a frequency $\omega = 2\pi\mathtt{f} = 2\pi(\mathtt{60})$. The visualization of the active and reactive power signals of the complex-based model is obtained from the expression given in (4.7) with the functions `real` e `imag` applied to the solution of the apparent power expressed in complexes.

Using the Matlab script (Section 7.1.3) in open loop we obtain the graphs referring to active power, reactive power and frequency of the two inverters of the MG as shown in the figure 30.



(a) Active Power $P_{(3\phi)}$ (b) Reactive Power $Q_{(3\phi)}$ (c) Frequency ω

Figure 30: Complex-based MG model system dynamics when two three-phase inverters with equal voltage magnitude and equal phases operate during the simulation time.

When inverter 2 delivers an active power flow (P_2) greater than inverter 1 (P_1) in the MG (subfigure 30a), the reactive power flow of inverter 2 (Q_2) is less than that supplied by inverter 1 (Q_1) (subfigure 30b). The frequency remains constant due to open-loop analysis (subfigure 30c).

If we focus on the active power signal supplied by the inverters to the resistive load, as shown in the figure 31, we can verify that this signal corresponds to the active power signal obtained with the MNA-based MG model shown in figure 26, considering that we use the same parameters to simulate the two models.

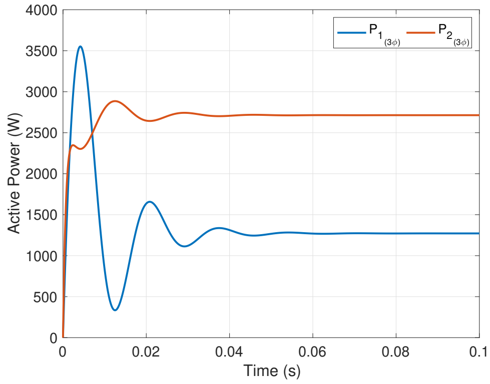


Figure 31: Active Power $P_{(3\phi)}$ of complex-based MG model

4.4 Valoration

The expression in (4.14) or in the state space form (4.15) of the complex-based MG model gives us the opportunity to observe the interaction of the components within the system, i.e., how the apparent power of each inverter is related to the voltages and phases of each of them.

The MG modeling procedure using complex-based manual analysis is reduced, unlike the modeling procedure reviewed in the previous sections using MNA-based manual analysis.

It is also important to mention the reduction in the number of algorithmic procedures and Matlab functions when we use complex-based modeling, where we relatively observe a reduction of programming code for analysis and simulation that can be reviewed in the appendix of this document.

5 Stability analysis and control of MG composed by two three-phase inverters feeding a resistive load in islanded mode

In this last section we are going to perform the stability analysis of the complex-based model using Matlab to obtain the equilibrium points considering the parameters of the table 11. In addition, the droop control strategy will be implemented in the complex-based model and in the MNA-based model, to observe the power delivered by the MG inverters in closed-loop and value the respective simulations between the two models.

5.1 Equilibrium points of complex-based MG three-phase model

The equilibrium points or trajectories of a nonlinear system are obtained by solving the equation $\dot{x} = \frac{dx}{dt} = 0$ where x is the state variable given the expression [15]

$$\dot{x}(t) = f(x(t), u(t)), \quad x(t_0) = x_0 \tag{5.1}$$

$$y(t) = h(x(t)) \tag{5.2}$$

As cited above, we proceed to calculate the equilibrium points of the system of equations of the complex-based model using Matlab, given the equations in (4.14) as follows

$$\begin{aligned} \dot{S}_1(t) &= 0 \\ \dot{S}_2(t) &= 0 \end{aligned} \tag{5.3}$$

so, we have

$$\begin{aligned} \dot{S}_1(t) &= \left(\frac{\dot{E}_1(t)}{E_1(t)} + j(\omega_0 + \dot{\varphi}_1(t)) - \frac{(R_L + R_1)}{L_1} \right) S_1(t) + \frac{3}{2}\frac{E_1^2(t)}{L_1} - \frac{R_L}{L_1} S_2(t)\alpha = 0 \\ \dot{S}_2(t) &= \left(\frac{\dot{E}_2(t)}{E_2(t)} + j(\omega_0 + \dot{\varphi}_2(t)) - \frac{(R_L + R_2)}{L_2} \right) S_2(t) + \frac{3}{2}\frac{E_2^2(t)}{L_2} - \frac{R_L}{L_2} S_1(t)\alpha^{-1} = 0 \end{aligned} \tag{5.4}$$

Considering as null the rates of variation of voltage and phase with respect to time we have

$$\begin{aligned} \dot{S}_1(t) &= \left(j(\omega_0) - \frac{(R_L + R_1)}{L_1} \right) S_1(t) + \frac{3}{2}\frac{E_1^2(t)}{L_1} - \frac{R_L}{L_1} S_2(t)\alpha = 0 \\ \dot{S}_2(t) &= \left(j(\omega_0) - \frac{(R_L + R_2)}{L_2} \right) S_2(t) + \frac{3}{2}\frac{E_2^2(t)}{L_2} - \frac{R_L}{L_2} S_1(t)\alpha^{-1} = 0 \end{aligned} \tag{5.5}$$

where

$$\alpha = \frac{e_1(t)}{e_2(t)} = \frac{E1}{E2} e^{j(\varphi_1(t) - \varphi_2(t))} \tag{5.6}$$

Now we perform the calculations symbolically using the Matlab's script in the appendix section 7.1.4 obtaining the following equilibrium points

$$\begin{aligned} S1e &= \frac{E_1\left(E_1\,R_2 + E_1\,\mathrm{RL} - E_1\,L_2\,w\,1\mathrm{i} - E_2\,\mathrm{RL}\,\mathrm{e}^{\mathrm{AP}\,1\mathrm{i}}\right)3\mathrm{i}}{2\,(R_1\,R_2\,1\mathrm{i} + R_1\,\mathrm{RL}\,1\mathrm{i} + R_2\,\mathrm{RL}\,1\mathrm{i} + L_1\,R_2\,w + L_2\,R_1\,w + L_1\,\mathrm{RL}\,w + L_2\,\mathrm{RL}\,w - L_1\,L_2\,w^2\,1\mathrm{i})} \\ S2e &= \frac{E_2\left(E_2\,R_1\,1\mathrm{i} + E_2\,\mathrm{RL}\,1\mathrm{i} + E_2\,L_1\,w - E_1\,\mathrm{RL}\,\mathrm{e}^{-\mathrm{AP}\,1\mathrm{i}}\,1\mathrm{i}\right)3\mathrm{i}}{2\,(L_1\,L_2\,w^2 - R_1\,\mathrm{RL} - R_2\,\mathrm{RL} - R_1\,R_2 + L_1\,R_2\,w\,1\mathrm{i} + L_2\,R_1\,w\,1\mathrm{i} + L_1\,\mathrm{RL}\,w\,1\mathrm{i} + L_2\,\mathrm{RL}\,w\,1\mathrm{i})} \end{aligned} \tag{5.7}$$

considering the phase variation $AP = \varphi_1(t) - \varphi_2(t)$ in (5.7). At this point, it is interesting to ask ourselves if we can equalize the powers of the two MG inverters by manipulating only the voltages, evaluating the MG parameters from table 11 in (5.7). The results obtained in Matlab as follow

$$\begin{aligned} E_1 &= 0 \\ E_2 &= 0 \end{aligned} \tag{5.8}$$

where $E1$ and $E2$ are the output voltages of each inverter respectively. These voltages in (5.8) indicate that have to be zero, so that the powers of the two inverters are equal, therefore, there is no solution modifying only voltages.

When we equalize the powers of the two inverters of the MG, manipulating only the phases, we have a different case where we do not have an explicit solution because the phase relation depends implicitly on the voltages relation as shown in the expression (5.6) and (5.7), and the solutions in function of the real and imaginary part go according to the parameters of the MG, in this case, using the parameters in table 11, we have the following equilibrium points (S_e) simbolically

$$S_e = \begin{bmatrix} -\ln\left(\frac{\left(E_2{}^2\,L_1\,w - E_1{}^2\,L_2\,w - E_1{}^2\,R_2\,1\mathrm{i} + E_2{}^2\,R_1\,1\mathrm{i} - E_1{}^2\,\mathrm{RL}\,1\mathrm{i} + E_2{}^2\,\mathrm{RL}\,1\mathrm{i} + \sqrt{\left(E_1{}^2\,R_2 - E_2{}^2\,R_1 + E_1{}^2\,\mathrm{RL} - E_2{}^2\,\mathrm{RL} - E_1{}^2\,L_2\,w\,1\mathrm{i} + E_2{}^2\,L_1\,w\,1\mathrm{i}\right)^2 + 4\,E_1{}^2\,E_2{}^2\,\mathrm{RL}^2\,1\mathrm{i}}\right)1\mathrm{i}}{2\,E_1\,E_2\,\mathrm{RL}}\right)1\mathrm{i} \\ -\ln\left(-\frac{\left(E_1{}^2\,L_2\,w - E_2{}^2\,L_1\,w + E_1{}^2\,R_2\,1\mathrm{i} - E_2{}^2\,R_1\,1\mathrm{i} + E_1{}^2\,\mathrm{RL}\,1\mathrm{i} - E_2{}^2\,\mathrm{RL}\,1\mathrm{i} + \sqrt{\left(E_1{}^2\,R_2 - E_2{}^2\,R_1 + E_1{}^2\,\mathrm{RL} - E_2{}^2\,\mathrm{RL} - E_1{}^2\,L_2\,w\,1\mathrm{i} + E_2{}^2\,L_1\,w\,1\mathrm{i}\right)^2 + 4\,E_1{}^2\,E_2{}^2\,\mathrm{RL}^2\,1\mathrm{i}}\right)1\mathrm{i}}{2\,E_1\,E_2\,\mathrm{RL}}\right)1\mathrm{i} \end{bmatrix} \tag{5.9}$$

Executing the detailed script in appendix section 7.1.4 in Matlab, we have the complex power value as below

$$S_1e = S_2e = 1298.513 + j4489.058$$
$$S_1e = S_2e = 2651.940 + j837.341$$

with the values obtained in (5.9) respectively. Therefore, it is necessary to manipulate voltages and phases together to equalize the powers of the inverters of the complex-based MG model.

5.2 Droop control strategy of MNA-based and complex-based MG three-phase model

We are going to implement as a primary control loop the power sharing control in each inverter based on the droop control strategy, also called decentralized control. This type of control does not require communications service between MG inverters.

The droop control principle consists in coordinating the interaction of the frequency-active power droop characteristic and the voltage-reactive power droop characteristic, in such a way that it controls the flow of active and reactive power by controlling the frequency and amplitude of the output voltage in order to share the adjustment of the total power demand [16].

The general droop equations for the frequency and amplitude of the output voltage of an inverter are represented in the figure 32 and can be expressed as below

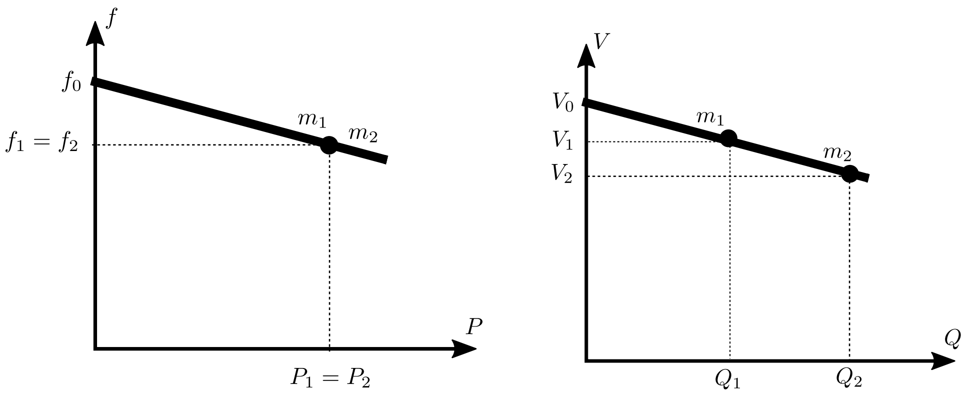
$$f = f_0 - m\Delta P \tag{5.10}$$
$$V = V_0 - n\Delta Q \tag{5.11}$$

where f_0 and V_0 are the reference values of the frequency and amplitude of the inverter output voltage respectively, ΔP is the change in active power and ΔQ is the variation of reactive power. Coefficients m y n are static-droop gains that can be adjusted as follows [16]

$$m = \frac{f_0 - f_{min}}{P_{max}} \tag{5.12}$$
$$n = \frac{V_0 - V_{min}}{Q_{max}} \tag{5.13}$$

where P_{max}, Q_{max} is the maximum active and reactive output power respectively, and f_{min}, $Vmin$ is the minimum output voltage and frequency respectively.

The droop control to share power in a MG composed by two inverters in parallel is represented in figure 32



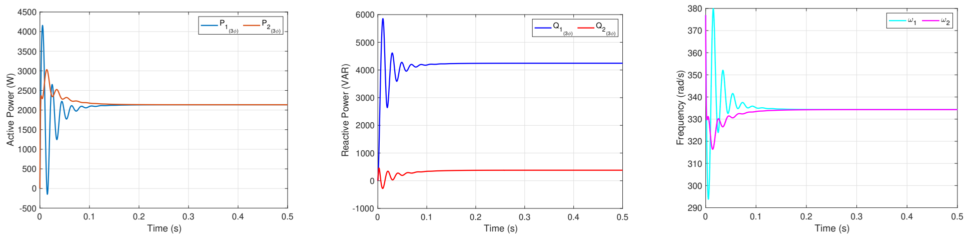
(a) $f - P$ Droop caracteristics (b) $V - Q$ Droop caracteristics

Figure 32: Relationship between frecuency control $(f - P)$ and voltage control $(V - Q)$ of Droop controller.

The simulation in closed-loop of the complex-based MG model composed by two three-phase inverters feeding a resistive load has been carried out algorithmically using a Matlab script in discrete similar to used in open-loop in section 4.3, but adding the droop equations as control loop.

A simulation time $\mathtt{t_f} = \mathtt{0.5}$ seconds has been configured, with steps $\mathtt{t_s} = 1\mathrm{e} - 6$, a frequency $\omega = 2\pi\mathtt{f} = 2\pi(60)$. The visualization of the active and reactive power signals of the complex-based model is obtained from the expression given in (4.7) with the functions `real` e `imag` applied to the solution of the apparent power expressed in complexes.

Using the Matlab script (Appendix Section 7.1.5) in closed-loop we obtain the graphs referring to active power, reactive power and frequency of the two inverters of complex-based MG model as shown in the figure 33.



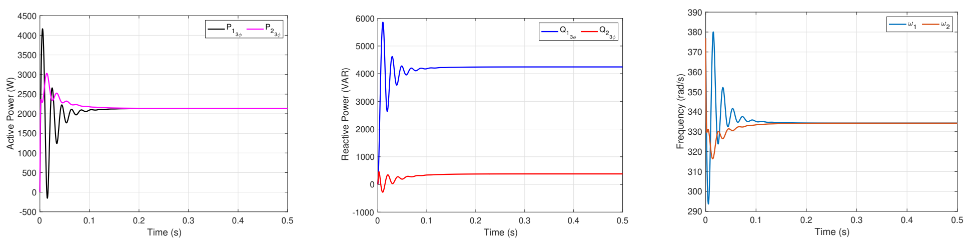
(a) Active Power $P_{(3\phi)}$ (b) Reactive Power $Q_{(3\phi)}$ (c) Frequency ω

Figure 33: Complex-based MG model system dynamics with Droop Control when two three-phase inverters with equal voltage magnitude and equal phases operate during the simulation time.

In the same way, the simulation in closed-loop of the MNA-based MG model composed by two three-phase inverters feeding a resistive load has been carried out algorithmically using a Matlab script in discrete similar to used in open-loop in section 3.4, but adding the droop equations as control loop.

So that the simulation results obtained with the two MG modeling methods can be properly compared, the same parameters have been adjusted as the simulation time $\mathtt{t_f} = 0.5$ seconds, steps $\mathtt{t_s} = 1e-6$ and frequency $\omega = 2\pi\mathtt{f} = 2\pi(60)$. The visualization of the active and reactive power signals of the MNA-based model is obtained from the product of the current and the voltage using functions `real` e `imag` applied to voltage expressed in complexes.

Using the Matlab script (Appendix Section 7.1.6) in closed-loop we obtain the graphs referring to active power, reactive power and frequency of the two inverters of MNA-based MG model as shown in the figure 34.



(a) Active Power $P_{(3\phi)}$ (b) Reactive Power $Q_{(3\phi)}$ (c) Frequency ω

Figure 34: MNA-based MG model system dynamics with Droop Control when two three-phase inverters with equal voltage magnitude and equal phases operate during the simulation time.

Relating the simulations of the MNA-based model and the complex-based model we can discuss that the subfigure 33a and subfigure 34a shows us the process of sharing the active power demand of the MG between inverter 1 (P_1) and inverter 2 (P_2) during the simulation time established in Matlab.

During the first $t = 0.1$ seconds we observe oscillations of the active power of the two inverters until reaching the total power balance required by the MG load. These transient oscillations can be attenuated by fine-tuning the primary controller parameters and implementing more levels of control in the models.

It has not been considered to pay greater attention to the reactive powers of the inverters represented in the subfigure 33b and subfigure 34b, because the droop primary control has been enough to understand the principle of its control within the MNA-based MG model and complex-based MG model feeding a resistive load proposed in the present work.

Besides, the subfigure 33c and subfigure 34c illustrates the frequency reduction in the inverters when they reach the supply balance of active power demanded by the load. To avoid having this unwanted effect on frequency, it is necessary to increase the control levels in the MG model.

6 Conclusions

Through the analysis of a basic electrical circuit, we have understood the principle of the MNA method to analyze more complex electrical circuits, providing more information than the mesh voltages method or the node currents method. In this case, it has allowed us to model an MG composed by two single-phase inverters and another MG composed of two three-phase inverters that feed a resistive load.

The MNA-based MG model is easily automatable using Matlab computational algorithms, which reduces the time spent on manual mathematical analysis. The MNA-based SCAM tool is developed as a Matlab script and has made it easier for us to model the MGs mentioned above in a fast, simple and scalable way to larger MGs. The results obtained by SCAM have been compared to the analytical results obtained manually, obtaining the same results in less time.

To simulate the dynamic response of the MG models analyzed in this work, it was necessary to transform the result generated by the SCAM tool given in the Transfer Function to State Space. This managed to algorithmically improve the analysis and simulation process in Matlab, allowing to obtain the value of the power of the MG inverters by making the product of the current and the output voltage of each one of them. We avoid the analysis in the Transfer Function model because it leads to other additional mathematical operations that include the integral of the convolution of the product of the current and the voltage to obtain power, which was not part of the objectives.

When MGs increase in number of inverters, the analysis using methods such as MNA becomes very extensive and makes it difficult to understand the behavior of the system. The complex-based MG model is a very good alternative to the MNA-based MG models, due to its reduced mathematical load, facility of intuition and observation of the interaction of MG components compared to the other models reviewed.

The algorithmic structure of the complex-based MG model implemented in Matlab is smaller compared to the structure of the MNA-based model, which provides greater fluidity in simulation tests. The validity and reliability of the complex-based model is checked with the results of the simulations carried out, where the power signal has a similar behavior in both cases.

Finally, the droop control strategy was implemented to give greater strength to the complex-based MG model approach. In this case, the result of the stability analysis indicates that it is necessary to jointly manipulate magnitude and phase to equalize the powers generated by the inverters and achieve the shared power balance that the load demands in the MG. In view of the results obtained with the simulations, the dynamic response of the proposed models has similar behaviors, giving validity and confidence to the complex-based MG model, and allowing future works in this line.

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7 Appendix

7.1 Matlab Scripts

7.1.1 MNA-based MG single-phase model Matlab script

```
clc;
clear all;
fname="ModeloMicrorredElectrico_1FASE.cir";
scam;

R1 = 3;
R2 = 3;
L1 = 0.01;
L2 = 0.01;
RL = 1000;

G1 = -I_V1
G2 = -I_V2

G1 = collect(G1,[V1 V2])
G2 = collect(G2,[V1 V2])

G11 = coeffs(G1,V1)
G11 = G11(1,2)

G12 = coeffs(G1,V2)
G12 = G12(1,2)

G21 = coeffs(G2,V1)
G21 = G21(1,2)

G22 = coeffs(G2,V2)
G22 = G22(1,2)

pretty([G11 G12;G21 G22])

G=subs([G11 G12;G21 G22])

pretty(G)

% Make Generic Transfer Function Simple
s = tf('s');
```

```
[rows,columns]=size(G);% rows and columns of
   matrix G (Various Func. Transf)
Gtf = tf(zeros(size(G)));
for fil=1:rows
  for col=1:columns
  Gtf(fil,col)=eval(char(G(fil,col)));
  end
end

% First way to obtain State Spaces matrices (ss)
Gss = ss(Gtf)
A = Gss.A;
B = Gss.B;
C = Gss.C;
D = Gss.D;

%% Simulation using Euler - FOR Loop (Fast with
   preallocating zeros)
tic
close all;
tf = 0.1; % simulation end time
ts = 1e-6;  % simulation steps: ts = 1 / 4. * p
   where p: poles -> eig (A)
t = 0:ts:tf; % time range vector
x = [0 ; 0 ; 0 ; 0]; %State Initial Values Vector
Estados = zeros(length(x),length(t)-1); %Vector of
    zeros assigned to states
U = zeros(2,length(t)-1); %Zeros vector assigned
   to Inputs
Salidas = zeros(2,length(t)-1); %Zeros vector
   assigned to Outputs
w=2*pi*60; % value as a function of frequency
ph1=0:0.0001:0.001;ph2=0:0.0001:0.001; % phase
   range
P1rms = zeros(1,length(ph2)); %Vector of zeros
   assigned to Power 1 rms
P2rms = zeros(1,length(ph2));%Vector of zeros
   assigned to Power 2 rms

for sim = 1:length(ph2) %number iterations
   according to the phase
  for k = 1:length(t)-1 %number iterations
     according to time
```

```
    u1 = 155*sin(w*k*ts+ph1(1));% Input 1 voltage
       value
    u2 = 155*sin(w*k*ts+ph2(sim));% Input 2 voltage
       value
    u = [u1 ; u2];% inputs vector

    %STATE EQUATION
    x = x + ts*(A*x + B*u); % Euler: x(k+1) = x(k) +
        ts*(dx/dt)
    % where:    dx/dt = A*x + B*u

    %OUTPUT EQUATION
    I = C*x;

    Estados(:,k) = x; % Values of state variables
    U(:,k) = u; % Input values (voltage)
    Salidas(:,k) = I; % Values of output variables (
       Current)

    end

    P = Salidas.*U; %Potencia Activa (Potencia=
       Corriente.*Tension)

    P1rms(:,sim) = rms(P(1,:)); %RMS Power 1
    P2rms(:,sim) = rms(P(2,:)); %RMS Power 2

      if (sim == 1) || (sim == length(ph2))

      figure();
      plot((0:length(t)-2)*ts,Salidas(1,:),'b',(0:
         length(t)-2)*ts,Salidas(2,:),'r--','
         LineWidth',2);
      %title('Outputs');
      xlabel('Time␣(s)'); ylabel('Current␣(A)');
      legend('I_1','I_2','Orientation','horizontal')
         ;
      grid on;axis fill;set(gca,'Fontsize',15);
      if sim == 1, saveas(gcf,'
         corrientes_primera_viguales','epsc'); end
      if sim == length(ph2), saveas(gcf,'
         corrientes_ultima_viguales','epsc'); end
      figure();
```

```
    plot((0:length(t)-2)*ts,U(1,:),(0:length(t)-2)
       *ts,U(2,:),'--','LineWidth',2);
    %title('Inputs');
    xlabel('Time␣(s)'); ylabel('Voltage␣(V)');
    legend('V_1','V_2','Orientation','horizontal')
       ;
    grid on;axis fill;set(gca,'Fontsize',15);
    if sim == 1, saveas(gcf,'
       voltajes_primera_viguales','epsc'); end
    if sim == length(ph2), saveas(gcf,'
       voltajes_ultima_viguales','epsc'); end
    figure();
    plot((0:length(t)-2)*ts,P(1,:),'g',(0:length(t
       )-2)*ts,P(2,:),'b--','LineWidth',2);
    %title('Powers');
    xlabel('Time␣(s)'); ylabel('Power␣(W)');
    legend('P_1','P_2','Orientation','horizontal')
       ;
    grid on;axis fill;set(gca,'Fontsize',15);
    if sim == 1, saveas(gcf,'
       potencias_primera_viguales','epsc'); end
    if sim == length(ph2), saveas(gcf,'
       potencias_ultima_viguales','epsc'); end
    end
end
figure();
plot(ph2,single(P1rms),ph2,single(P2rms),'--','
   LineWidth',2);
%title('(RMS) Active Power ');
xlabel('\phi_{2}(phase)','Fontsize',15); ylabel('
   Power_{RMS}␣(W)','Fontsize',15);
legend('P_1␣_{RMS}','P_2␣_{RMS}','Orientation','
   horizontal','FontSize',15);
grid('on');axis fill; set(gca,'Fontsize',15);
saveas(gcf,'potencia_rms_viguales','epsc')
toc
```

7.1.2 MNA-based MG three-phase model Matlab script

```
clc;
clear all;
fname="ModeloMicrorredElectrico_3FASE.cir";
scam;
```

```
R1a = 0.5;
R1b = 0.5;
R1c = 0.5;
R2a = 10;
R2b = 10;
R2c = 10;
L1a = 2e-2;
L1b = 2e-2;
L1c = 2e-2;
L2a = 1e-2;
L2b = 1e-2;
L2c = 1e-2;
RLa = 2;
RLb = 2;
RLc = 2;

G1a = -I_V1a
G1b = -I_V1b
G1c = -I_V1c
G2a = -I_V2a
G2b = -I_V2b
G2c = -I_V2c

G1a = collect(G1a,[V1a V2a])
G1b = collect(G1b,[V1b V2b])
G1c = collect(G1c,[V1c V2c])
G2a = collect(G2a,[V1a V2a])
G2b = collect(G2b,[V1b V2b])
G2c = collect(G2c,[V1c V2c])
%%%%%%%%%%%%%%%%%%%%%%%
G11 = coeffs(G1a,V1a)
G11 = G11(1,2)

%G12 = coeffs(G1a,V1b)
G12 = 0;

%G13 = coeffs(G1a,V1c)
G13 = 0;

G14 = coeffs(G1a,V2a)
G14 = G14(1,2)

%G15 = coeffs(G1a,V2b)
```

```
G15 = 0;

%G16 = coeffs(G1a,V2c)
G16 = 0;
%%%%%%%%%%%%%%%%%%%%%%%
%G21 = coeffs(G1b,V1a)
G21 = 0;

G22 = coeffs(G1b,V1b)
G22 = G22(1,2)

%G23 = coeffs(G1b,V1c)
G23 = 0;

%G24 = coeffs(G1b,V2a)
G24 = 0;

G25 = coeffs(G1b,V2b)
G25 = G25(1,2)

%G26 = coeffs(G1b,V2c)
G26 = 0;
%%%%%%%%%%%%%%%%%%%%%%
%G31 = coeffs(G1c,V1a)
G31 = 0;

%G32 = coeffs(G1c,V1b)
G32 = 0;

G33 = coeffs(G1c,V1c)
G33 = G33(1,2)

%G34 = coeffs(G1c,V2a)
G34 = 0;

%G35 = coeffs(G1c,V2b)
G35 = 0;

G36 = coeffs(G1c,V2c)
G36 = G36(1,2)
%%%%%%%%%%%%%%%%%%%%%%
G41 = coeffs(G2a,V1a)
G41 = G41(1,2)
```

```
%G42 = coeffs(G2a,V1b)
G42 = 0;

%G43 = coeffs(G2a,V1c)
G43 = 0;

G44 = coeffs(G2a,V2a)
G44 = G44(1,2)

%G45 = coeffs(G2a,V2b)
G45 = 0;

%G46 = coeffs(G2a,V2c)
G46 = 0;
%%%%%%%%%%%%%%%%%%%%%%
%G51 = coeffs(G2b,V1a)
G51 = 0;

G52 = coeffs(G2b,V1b)
G52 = G52(1,2)

%G53 = coeffs(G2b,V1c)
G53 = 0;

%G54 = coeffs(G2b,V2a)
G54 = 0;

G55 = coeffs(G2b,V2b)
G55 = G55(1,2)

%G56 = coeffs(G2b,V2c)
G56 = 0;
%%%%%%%%%%%%%%%%%%%%%%
%G61 = coeffs(G2c,V1a)
G61 = 0;

%G62 = coeffs(G2c,V1b)
G62 = 0;

G63 = coeffs(G2c,V1c)
G63 = G63(1,2)

%G64 = coeffs(G2c,V2a)
G64 = 0;
```

```
%G65 = coeffs(G2c,V2b)
G65 = 0;

G66 = coeffs(G2c,V2c)
G66 = G66(1,2)
%%%%%%%%%%%%%%%%%%%%%

pretty([G11 G12 G13 G14 G15 G16 ;
G21 G22 G23 G24 G25 G26 ;
G31 G32 G33 G34 G35 G36 ;
G41 G42 G43 G44 G45 G46 ;
G51 G52 G53 G54 G55 G56 ;
G61 G62 G63 G64 G65 G66 ])

G=subs([G11 G12 G13 G14 G15 G16 ;
G21 G22 G23 G24 G25 G26 ;
G31 G32 G33 G34 G35 G36 ;
G41 G42 G43 G44 G45 G46 ;
G51 G52 G53 G54 G55 G56 ;
G61 G62 G63 G64 G65 G66 ])

pretty(G)

s = tf('s') % Make Generic Transfer Function
    Simple
[filas,columnas]=size(G);% rows and columns of
    matrix G (Various Func. Transf)
Gtf = tf(zeros(size(G)));
for fil=1:filas
  for col=1:columnas
  Gtf(fil,col)=eval(char(G(fil,col)));
  end
end

% First way to obtain State Spaces matrices (ss)
Gss = ss(Gtf)
A = Gss.A;
B = Gss.B;
C = Gss.C;
D = Gss.D;

%% Simulation using Euler - FOR Loop (Fast with
    preallocating zeros)
```

```
tic
close all;
tf = 0.1; % simulation end time
ts = 1e-6;  % simulation steps: ts=1/4.*(p) where
   p: poles -> eig(A)
t = 0:ts:tf; % time range vector
x = [0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ;
   0];%State Initial Values Vector
Estados = zeros(length(x),length(t)-1); %Vector of
    zeros assigned to states
U = zeros(6,length(t)-1); %Zeros vector assigned
   to Inputs
Salidas = zeros(6,length(t)-1); %Zeros vector
   assigned to Outputs
Wr = zeros(6,length(t)-1); %Zeros vector assigned
   to Frequency
w0=2*pi*60; % value as a function of frequency
u0=155; % Initial voltage magnitude
phase=2*pi/3; %Phase=120
ph1=0; % Input 1 phase range
ph2=0:0.05:0.5; % Input 2 phase range
opt=1;%opt=2;

%P1_3frms = zeros(1,length(ph2));
%P2_3frms = zeros(1,length(ph2));
for sim = 1:length(ph2) %number iterations
   according to the phase
  for k = 1:length(t)-1 %number iterations
     according to time

  u1a = real(u0*exp(1i*(w0*k*ts)+1i*(0+ph1))); %
     Input 1a voltage value
  u1b = real(u0*exp(1i*(w0*k*ts)+1i*(phase+ph1)));
      % Input 1b voltage value
  u1c = real(u0*exp(1i*(w0*k*ts)+1i*(2*phase+ph1))
     ); % Input 1c voltage value

  u2a = real(u0*exp(1i*(w0*k*ts)+1i*(0+ph2(sim))))
     ; % Input 2a voltage value
  u2b = real(u0*exp(1i*(w0*k*ts)+1i*(phase+ph2(sim
     )))); % Input 2b voltage value
  u2c = real(u0*exp(1i*(w0*k*ts)+1i*(2*phase+ph2(
     sim)))); % Input 2c voltage value
```

```
    u = [u1a ; u1b ; u1c ; u2a ; u2b ; u2c]; %
       inputs vector

    %STATE EQUATION
    x = x + ts*(A*x + B*u); % Euler: x(k+1) = x(k) +
        ts*(dx/dt)
    % where:   dx/dt = A*x + B*u

    %OUTPUT EQUATION
    I = C*x;

    Estados(:,k) = x; % Values of state variables
    U(:,k) = u; % Input values (voltage)
    Salidas(:,k) = I; % Values of output variables (
       Current)
    Wr(:,k) = w0; %Values of frequency variables
    end


    P = Salidas.*U; %Active Power (Power=Current.*
       Voltage)

    P1_3f = P(1,:)+P(2,:)+P(3,:); %Three Phase
       Active Power 1
    P2_3f = P(4,:)+P(5,:)+P(6,:); %Three Phase
       Active Power 2

    if (sim == 1) || (sim == length(ph2))

      if opt == 1
    %%%%%%%%%--INDIVIDUAL PLOT::V1--%%%%%%%%
  figure();
  plot((0:length(t)-2)*ts,Salidas(1,:),(0:length(t)
     -2)*ts,Salidas(2,:),(0:length(t)-2)*ts,Salidas
     (3,:),'LineWidth',2);
  %title('Outputs');
  xlabel('Time␣(s)'); ylabel('Current␣(A)');
  legend('I_{1a}','I_{1b}','I_{1c}','Orientation','
     horizontal');
  grid on;axis fill;set(gca,'Fontsize',15);
  if sim == 1, saveas(gcf,'3
     f_v1_corrientes_primera_viguales_desfase','epsc
     '); end
```

```
if sim == length(ph2), saveas(gcf,'3
    f_v1_corrientes_ultima_viguales_desfase','epsc'
    ); end
figure();
plot((0:length(t)-2)*ts,U(1,:),'r-',(0:length(t)
    -2)*ts,U(2,:),'b-',(0:length(t)-2)*ts,U(3,:),'g
    -','LineWidth',2);
%title('Inputs');
xlabel('Time␣(s)'); ylabel('Voltage␣(V)');
legend('V_{1a}','V_{1b}','V_{1c}','Orientation','
    horizontal');
grid on;axis fill;set(gca,'Fontsize',15);
if sim == 1, saveas(gcf,'3
    f_v1_voltajes_primera_viguales_desfase','epsc')
    ; end
if sim == length(ph2), saveas(gcf,'3
    f_v1_voltajes_ultima_viguales_desfase','epsc');
     end
figure();
plot((0:length(t)-2)*ts,P1_3f,'k-',(0:length(t)-2)
    *ts,P2_3f,'m-','LineWidth',2);
%title('Powers');
xlabel('Time␣(s)'); ylabel('Active␣Power␣(W)');
legend('P_{1␣3\phi}','P_{2␣3\phi}','Orientation','
    horizontal');
grid on;axis fill;set(gca,'Fontsize',15);
if sim == 1, saveas(gcf,'3
    f_potencias_primera_viguales_desfase','epsc');
    end
if sim == length(ph2), saveas(gcf,'3
    f_potencias_ultima_viguales_desfase','epsc');
    end
%%%%%%%%%%%%%%%%%%%
    end
    if opt == 2
%%%%%%%%%--INDIVIDUAL PLOT::V2--%%%%%%%%%%%
figure();
plot((0:length(t)-2)*ts,Salidas(4,:),(0:length(t)
    -2)*ts,Salidas(5,:),(0:length(t)-2)*ts,Salidas
    (6,:),'LineWidth',2);
%title('Outputs');
xlabel('Time␣(s)'); ylabel('Current␣(A)');
legend('I_{2a}','I_{2b}','I_{2c}','Orientation','
    horizontal');
```

```
grid on;axis fill;set(gca,'Fontsize',15);
if sim == 1, saveas(gcf,'3
   f_v2_corrientes_primera_viguales_desfase','epsc
   '); end
if sim == length(ph2), saveas(gcf,'3
   f_v2_corrientes_ultima_viguales_desfase','epsc'
   ); end
figure();
plot((0:length(t)-2)*ts,U(4,:),'r-',(0:length(t)
   -2)*ts,U(5,:),'b-',(0:length(t)-2)*ts,U(6,:),'g
   -','LineWidth',2);
%title('Inputs');
xlabel('Time␣(s)'); ylabel('Voltage␣(V)');
legend('V_{2a}','V_{2b}','V_{2c}','Orientation','
   horizontal');
grid on;axis fill;set(gca,'Fontsize',15);
if sim == 1, saveas(gcf,'3
   f_v2_voltajes_primera_viguales_desfase','epsc')
   ; end
if sim == length(ph2), saveas(gcf,'3
   f_v2_voltajes_ultima_viguales_desfase','epsc');
    end
figure();
plot((0:length(t)-2)*ts,P1_3f,'k-',(0:length(t)-2)
   *ts,P2_3f,'m-','LineWidth',2);
%title('Powers');
xlabel('Time␣(s)'); ylabel('Active␣Power␣(W)');
legend('P_{1␣3\phi}','P_{2␣3\phi}','Orientation','
   horizontal');
grid on;axis fill;set(gca,'Fontsize',15);
if sim == 1, saveas(gcf,'3
   f_potencias_primera_viguales_desfase','epsc');
   end
if sim == length(ph2), saveas(gcf,'3
   f_potencias_ultima_viguales_desfase','epsc');
   end
%%%%%%%%%%%%%%%%%%
    end
  end
end
toc
```

7.1.3 Complex-based MG three-phase model Matlab script

```
tic
clc;
close all;
clear all;

R1=0.5;
R2=10;
RL=2;
L1=2e-2;
L2=1e-2;


step=1e-6;% simulation steps
t_fin=0.1; % simulation end time
steps=ceil(t_fin/step);% time range vector
S1=zeros(1,steps);%Vector of zeros assigned to Power
    1
S2=zeros(1,steps);%Vector of zeros assigned to Power
    2
wr=zeros(2,steps);%Vector of zeros assigned to
   Frequency

t=0;%Initial time
droop1=0;droop2=0;%Droop control selector (ON/OFF)
m=1e-3;n=0*1e-3;%Static-droop gains
s1=0;%Initial Power 1
s2=0;%Initial Power 2
E1=155;E2=155;E0=155;%Nominal Voltage
w1=2*pi*60;w2=2*pi*60;w0=2*pi*60;%Nominal Frequency
e1=E1*exp(1i*(w1*t));%Voltage 1 equation
e2=E2*exp(1i*(w2*t));%Voltage 2 equation
dw1=0;dw2=0;%Frequency variation
dE1=0;dE2=0;%Voltage variation
randomizator=@() rand(1,2)>=0.5
ph1=0;ph2=0;%Initial phase

for i=1:steps

S1(i)=s1;
S2(i)=s2;
E2_old=E2;E1_old=E1;
ph1_old=ph1;
```

```
ph2_old=ph2;
t=t+step;
if (mod(t,100e-6)<step)

real(s1)
real(s2)

end

wr(:,i)=[w1;w2]; % Values of Frequency 1 & 2

ph1=ph1+w1*step; %Values of phase 1
ph2=ph2+w2*step; %Values of phase 2
e1=E1*exp(1i*(ph1)); %Values of voltage 1
e2=E2*exp(1i*(ph2)); %Values of voltage 2
dp1=(ph1-ph1_old)/step; %Values of phase variation 1
dp2=(ph2-ph2_old)/step; %Values of phase variation 2
dE2=(E2-E2_old)/step; %Values of voltage variation 2
dE1=(E1-E1_old)/step; %Values of voltage variation 1
%Complex-based equations system
ds1=(dE1/E1-(RL+R1)/L1+1i*(dp1))*s1+(3/2)*E1^2/L1-RL
   /L1*s2*e1/e2;
ds2=(dE2/E2-(RL+R2)/L2+1i*(dp2))*s2+(3/2)*E2^2/L2-RL
   /L2*s1*e2/e1;

dE1=0;dE2=0; %zero voltage variation
s1=s1+ds1*step; %Values of Power 1
s2=s2+ds2*step; %Values of Power 2

end
%%%%%%%%--INDIVIDUAL PLOT--%%%%%%%%
figure('Name','Complex Model: Active Power','
   NumberTitle','off');
plot((0:steps-1)*step,real(S1),'LineWidth',2)
hold('on')
plot((0:steps-1)*step,real(S2),'LineWidth',2)
%title('Complex Model');
xlabel('Time (s)'); ylabel('Active Power (W)');
legend('P_{1_{(3\phi)}}','P_{2_{(3\phi)}}','
   Orientation','horizontal');
grid on;axis fill;set(gca,'Fontsize',15);
saveas(gcf,'
   complex_potencias_activ_prim_viguales_desfase','
   epsc');
```

```
hold off;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure('Name','Complex␣Model:␣Reactive␣Power','
   NumberTitle','off');
plot((0:steps-1)*step,imag(S1),'b-','LineWidth',2)
hold('on')
plot((0:steps-1)*step,imag(S2),'r-','LineWidth',2)
%title('Complex Model');
xlabel('Time␣(s)'); ylabel('Reactive␣Power␣(VAR)');
legend('Q_{1_{(3\phi)}}','Q_{2_{(3\phi)}}','
   Orientation','horizontal');
grid on;axis fill;set(gca,'Fontsize',15);
saveas(gcf,'
   complex_potencias_react_prim_viguales_desfase','
   epsc');
hold off;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure('Name','Complex␣Model:␣Frequency','
   NumberTitle','off');
plot((0:steps-1)*step,wr(1,:),'c-','LineWidth',2)
hold('on')
plot((0:steps-1)*step,wr(2,:),'m-','LineWidth',2)
%title('Complex Model');
xlabel('Time␣(s)'); ylabel('Frequency␣(rad/s)');
legend('\omega_1','\omega_2','Orientation','
   horizontal');
grid on;axis fill;set(gca,'Fontsize',15);
saveas(gcf,'complex_frec_prim_viguales_desfase','
   epsc');
hold off;
toc
```

7.1.4 Complex-based MG three-phase model equilibrium points Matlab script

```
syms dE1 dE2 E1 E2 dp1 dp2 t p1 p2 AP s1 s2 e1 e2
   R1 R2 RL L1 L2 w

dE1=0;dE2=0;%Voltage variation
dp1=0;dp2=0;%Phase variation
e1=E1*exp(1i*(w)*t+p1*1i);%Voltage 1
e2=E2*exp(1i*(w)*t+p2*1i);%Voltage 2
%AP=p1-p2;
```

```
ds1=(-(RL+R1)/L1+1i*w)*s1+(3/2)*E1^2/L1-RL/L1*s2*
    E1/E2*exp(1i*(AP)) == 0;
ds2=(-(RL+R2)/L2+1i*w)*s2+(3/2)*E2^2/L2-RL/L2*s1*
    E2/E1*exp(1i*(-AP)) == 0;

[s1e,s2e]=solve([ds1;ds2],[s1 s2])%Equilibrium
    Points Solve

Seqn1 = solve(s1e-s2e,[E1 E2])%Eq. Points
    depending on voltages

Seqn2 = solve(s1e-s2e,AP)%Eq. Points depending on
    phases

R1=0.5;%Resistor Inv. 1
R2=10;%Resistor Inv. 2
RL=2;%Resistor Load
L1=2e-2;%Inductor Inv. 1
L2=1e-2;%Inductor Inv. 2
E1=155;%Voltage magnitude 1
E2=155;%Voltage magnitude 2
w=2*pi*60;%phase->frecuency

vpa(subs(subs(s1e),AP,subs(Seqn2(1))))%simplify E.
    P 1 numeric sol. en Pot 1
vpa(subs(subs(s2e),AP,subs(Seqn2(1))))%simplify E.
    P 1 numeric sol. en Pot 2

vpa(subs(subs(s1e),AP,subs(Seqn2(2))))%simplify E.
    P 2 numeric sol. en Pot 1
vpa(subs(subs(s2e),AP,subs(Seqn2(2))))%simplify E.
    P 2 numeric sol. en Pot 2
```

7.1.5 Complex-based MG three-phase model droop control Matlab script

```
tic
clc;
close all;
clear all;

R1=0.5;
```

```
R2=10;
RL=2;
L1=2e-2;
L2=1e-2;


step=1e-6;% simulation steps
t_fin=0.5; % simulation end time
steps=ceil(t_fin/step);% time range vector
S1=zeros(1,steps);%Vector of zeros assigned to
    Power 1
S2=zeros(1,steps);%Vector of zeros assigned to
    Power 2
wr=zeros(2,steps);%Vector of zeros assigned to
    Frequency

t=0;%Initial time
droop1=1;droop2=1;%Droop control selector (ON/OFF)
m=1e-3;n=0*1e-3;%Static-droop gains
s1=0;%Initial Power 1
s2=0;%Initial Power 2
E1=155;E2=155;E0=155;%Nominal Voltage
w1=2*pi*60;w2=2*pi*60;w0=2*pi*60;%Nominal
    Frequency
e1=E1*exp(1i*(w1*t));%Voltage 1 equation
e2=E2*exp(1i*(w2*t));%Voltage 2 equation
dw1=0;dw2=0;%Frequency variation
dE1=0;dE2=0;%Voltage variation
randomizator=@() rand(1,2)>=0.5
ph1=0;ph2=0;%Initial phase

for i=1:steps

S1(i)=s1;
S2(i)=s2;
E2_old=E2;E1_old=E1;
ph1_old=ph1;
ph2_old=ph2;
t=t+step;
if (mod(t,100e-6)<step)

real(s1)
real(s2)
```

```
if droop1 %Droop control loop Inverter 1
E1=E0-n*(imag(s1)); %V-Q control 1
w1=w0-20*m*(real(s1)); %f-P control 1
end
if droop2 %Droop control loop Inverter 2
E2=E0-n*(imag(s2)); %V-Q control 2
w2=w0-20*m*(real(s2)); %f-P control 2
end

end

wr(:,i)=[w1;w2]; % Values of Frequency 1 & 2

ph1=ph1+w1*step; %Values of phase 1
ph2=ph2+w2*step; %Values of phase 2
e1=E1*exp(1i*(ph1)); %Values of voltage 1
e2=E2*exp(1i*(ph2)); %Values of voltage 2
dp1=(ph1-ph1_old)/step; %Values of phase variation
    1
dp2=(ph2-ph2_old)/step; %Values of phase variation
    2
dE2=(E2-E2_old)/step; %Values of voltage variation
    2
dE1=(E1-E1_old)/step; %Values of voltage variation
    1
%Complex-based equations system
ds1=(dE1/E1-(RL+R1)/L1+1i*(dp1))*s1+(3/2)*E1^2/L1-
   RL/L1*s2*e1/e2;
ds2=(dE2/E2-(RL+R2)/L2+1i*(dp2))*s2+(3/2)*E2^2/L2-
   RL/L2*s1*e2/e1;

dE1=0;dE2=0; %zero voltage variation
s1=s1+ds1*step; %Values of Power 1
s2=s2+ds2*step; %Values of Power 2

end
%%%%%%%%--INDIVIDUAL PLOT--%%%%%%%%
figure('Name','Complex␣Model:␣Active␣Power','
   NumberTitle','off');
plot((0:steps-1)*step,real(S1),'LineWidth',2)
hold('on')
plot((0:steps-1)*step,real(S2),'LineWidth',2)
%title('Complex Model');
xlabel('Time␣(s)'); ylabel('Active␣Power␣(W)');
```

```
legend('P_{1_{(3\phi)}}','P_{2_{(3\phi)}}','
   Orientation','horizontal');
grid on;axis fill;set(gca,'Fontsize',15);
saveas(gcf,'droop_comp_pot_activ_prim_viguales','
   epsc');
hold off;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure('Name','Complex␣Model:␣Reactive␣Power','
   NumberTitle','off');
plot((0:steps-1)*step,imag(S1),'b-','LineWidth',2)
hold('on')
plot((0:steps-1)*step,imag(S2),'r-','LineWidth',2)
%title('Complex Model');
xlabel('Time␣(s)'); ylabel('Reactive␣Power␣(VAR)')
   ;
legend('Q_{1_{(3\phi)}}','Q_{2_{(3\phi)}}','
   Orientation','horizontal');
grid on;axis fill;set(gca,'Fontsize',15);
saveas(gcf,'droop_comp_pot_react_prim_viguales','
   epsc');
hold off;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure('Name','Complex␣Model:␣Frequency','
   NumberTitle','off');
plot((0:steps-1)*step,wr(1,:),'c-','LineWidth',2)
hold('on')
plot((0:steps-1)*step,wr(2,:),'m-','LineWidth',2)
%title('Complex Model');
xlabel('Time␣(s)'); ylabel('Frequency␣(rad/s)');
legend('\omega_1','\omega_2','Orientation','
   horizontal');
grid on;axis fill;set(gca,'Fontsize',15);
saveas(gcf,'droop_comp_frec_prim_viguales','epsc')
   ;
hold off;
toc
```

7.1.6 MNA-based MG three-phase model droop control Matlab script

```
clc;
clear all;
fname="ModeloMicrorredElectrico_3FASE.cir";
```

```
scam;

R1a = 0.5;
R1b = 0.5;
R1c = 0.5;
R2a = 10;
R2b = 10;
R2c = 10;
L1a = 2e-2;
L1b = 2e-2;
L1c = 2e-2;
L2a = 1e-2;
L2b = 1e-2;
L2c = 1e-2;
RLa = 2;
RLb = 2;
RLc = 2;

G1a = -I_V1a
G1b = -I_V1b
G1c = -I_V1c
G2a = -I_V2a
G2b = -I_V2b
G2c = -I_V2c

G1a = collect(G1a,[V1a V2a])
G1b = collect(G1b,[V1b V2b])
G1c = collect(G1c,[V1c V2c])
G2a = collect(G2a,[V1a V2a])
G2b = collect(G2b,[V1b V2b])
G2c = collect(G2c,[V1c V2c])
%%%%%%%%%%%%%%%%%%%%%%%%
G11 = coeffs(G1a,V1a)
G11 = G11(1,2)

%G12 = coeffs(G1a,V1b)
G12 = 0;

%G13 = coeffs(G1a,V1c)
G13 = 0;

G14 = coeffs(G1a,V2a)
G14 = G14(1,2)
```

```
%G15 = coeffs(G1a,V2b)
G15 = 0;

%G16 = coeffs(G1a,V2c)
G16 = 0;
%%%%%%%%%%%%%%%%%%%%%%%
%G21 = coeffs(G1b,V1a)
G21 = 0;

G22 = coeffs(G1b,V1b)
G22 = G22(1,2)

%G23 = coeffs(G1b,V1c)
G23 = 0;

%G24 = coeffs(G1b,V2a)
G24 = 0;

G25 = coeffs(G1b,V2b)
G25 = G25(1,2)

%G26 = coeffs(G1b,V2c)
G26 = 0;
%%%%%%%%%%%%%%%%%%%%%%
%G31 = coeffs(G1c,V1a)
G31 = 0;

%G32 = coeffs(G1c,V1b)
G32 = 0;

G33 = coeffs(G1c,V1c)
G33 = G33(1,2)

%G34 = coeffs(G1c,V2a)
G34 = 0;

%G35 = coeffs(G1c,V2b)
G35 = 0;

G36 = coeffs(G1c,V2c)
G36 = G36(1,2)
%%%%%%%%%%%%%%%%%%%%%%
G41 = coeffs(G2a,V1a)
G41 = G41(1,2)
```

```
%G42 = coeffs(G2a,V1b)
G42 = 0;

%G43 = coeffs(G2a,V1c)
G43 = 0;

G44 = coeffs(G2a,V2a)
G44 = G44(1,2)

%G45 = coeffs(G2a,V2b)
G45 = 0;

%G46 = coeffs(G2a,V2c)
G46 = 0;
%%%%%%%%%%%%%%%%%%%%%%%
%G51 = coeffs(G2b,V1a)
G51 = 0;

G52 = coeffs(G2b,V1b)
G52 = G52(1,2)

%G53 = coeffs(G2b,V1c)
G53 = 0;

%G54 = coeffs(G2b,V2a)
G54 = 0;

G55 = coeffs(G2b,V2b)
G55 = G55(1,2)

%G56 = coeffs(G2b,V2c)
G56 = 0;
%%%%%%%%%%%%%%%%%%%%%%%
%G61 = coeffs(G2c,V1a)
G61 = 0;

%G62 = coeffs(G2c,V1b)
G62 = 0;

G63 = coeffs(G2c,V1c)
G63 = G63(1,2)

%G64 = coeffs(G2c,V2a)
```

```
G64 = 0;

%G65 = coeffs(G2c,V2b)
G65 = 0;

G66 = coeffs(G2c,V2c)
G66 = G66(1,2)
%%%%%%%%%%%%%%%%%%%%%

pretty([G11 G12 G13 G14 G15 G16 ;
G21 G22 G23 G24 G25 G26 ;
G31 G32 G33 G34 G35 G36 ;
G41 G42 G43 G44 G45 G46 ;
G51 G52 G53 G54 G55 G56 ;
G61 G62 G63 G64 G65 G66 ])

G=subs([G11 G12 G13 G14 G15 G16 ;
G21 G22 G23 G24 G25 G26 ;
G31 G32 G33 G34 G35 G36 ;
G41 G42 G43 G44 G45 G46 ;
G51 G52 G53 G54 G55 G56 ;
G61 G62 G63 G64 G65 G66 ])

pretty(G)

s = tf('s') % Make Generic Transfer Function
    Simple
[filas,columnas]=size(G);% rows and columns of
    matrix G (Various Func. Transf)
Gtf = tf(zeros(size(G)));
for fil=1:filas
for col=1:columnas
Gtf(fil,col)=eval(char(G(fil,col)));
end
end

% First way to obtain State Spaces matrices (ss)
Gss = ss(Gtf)
A = Gss.A;
B = Gss.B;
C = Gss.C;
D = Gss.D;
```

```
%% Simulation using Euler - FOR Loop (Fast with
   preallocating zeros)
tic
close all;
tf = 0.5; % simulation end time
ts = 1e-6;  % simulation steps: ts=1/4.*(p) where
   p: poles -> eig(A)
steps=ceil(tf/ts);% time range vector
x = [0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ;
   0];%State Initial Values Vector
Estados = zeros(length(x),steps); %Vector of zeros
    assigned to states
U = zeros(6,steps); %Zeros vector assigned to
   Inputs
Salidas = zeros(6,steps); %Zeros vector assigned
   to Outputs
w0=2*pi*60; % nominal value as a function of
   frequency
u0=155; % nominal voltage magnitude
phase=2*pi/3; %Phase 120
m=1e-3;n=0*1e-3;
w1=w0;w2=w0;%initial frequency 1 & 2
u1=u0;u2=u0;%initial voltage 1 & 2
Ps = zeros(2,steps); %Zeros vector assigned to
   Active Power
Qs = zeros(2,steps); %Zeros vector assigned to
   Reactive Power
Us = zeros(2,steps); %Zeros vector assigned to
   Inputs
W = zeros(2,steps); %Zeros vector assigned to
   Frequency
tsim=0;%Initial simulation time
[Ad,Bd]=c2d(A,B,ts);%Continuous2Discrete
   Conversion
ph1=0; % initial phase 1
ph2=0; % initial phase 2
opt=1;%1 or 2;%Selector graphics inverter 1 & 2
droop=1;%Selector droop control ON/OFF

for k = 1:steps %number iterations according to
   time

tsim=tsim+ts;
```

```
ph1=ph1+w1*ts; %Values of phase 1
ph2=ph2+w2*ts; %Values of phase 2

u1a = (u1*exp(1i*(ph1)+1i*(0))); % Input 1a
    voltage value
u1b = (u1*exp(1i*(ph1)+1i*(phase))); % Input 1b
    voltage value
u1c = (u1*exp(1i*(ph1)+1i*(2*phase))); % Input 1c
    voltage value

u2a = (u2*exp(1i*(ph2)+1i*(0))); % Input 2a
    voltage value
u2b = (u2*exp(1i*(ph2)+1i*(phase))); % Input 2b
    voltage value
u2c = (u2*exp(1i*(ph2)+1i*(2*phase))); % Input 2c
    voltage value

u = ([u1a ; u1b ; u1c ; u2a ; u2b ; u2c]);

%STATE EQUATION:
%Continuous:
%x = x + ts*(A*x + B*real(u)); % Euler: x(k+1) = x
    (k) + ts*(dx/dt) where:  dx/dt = A*x + B*u
%Discrete:
x = Ad*x + Bd*real(u);

%OUTPUT EQUATION
I = C*x;

p1=I(1:3)'*real(u(1:3));%Three-phase Active Power
    1
p2=I(4:6)'*real(u(4:6));%Three-phase Active Power
    2
q1 = I(1:3)'*imag(u(1:3));%Three-phase Reactive
    Power 1
q2 = I(4:6)'*imag(u(4:6));%Three-phase Reactive
    Power 2
Ps(:,k)=[p1;p2];%Active Power vector 1 & 2
Qs(:,k)=[q1;q2];%Reactive Power vector 1 & 2

if droop %Droop Control Loop
w1= w0 - 20*m*p1;  %f-P control 1
w2= w0 - 20*m*p2;  %f-P control 2
u1 = u0 - n*q1; %V-Q control 1
```

```
u2 = u0 - n*q2; %V-Q control 2
end

Estados(:,k) = x; % Values of state variables
U(:,k) = real(u); % Input values (voltages)
Us(:,k) = [u1;u2]; % Input values (voltage 1 & 2)
Salidas(:,k) = I; % Values of output variables (
   Currents)
W(:,k) = [w1;w2]; %Values of frequency variables
end
if opt == 1
%%%%%%%%--INDIVIDUAL PLOT::V1--%%%%%%%%
figure();
plot((0:steps-1)*ts,Salidas(1,:),(0:steps-1)*ts,
   Salidas(2,:),(0:steps-1)*ts,Salidas(3,:),'
   LineWidth',2);
%title('Outputs');
xlabel('Time␣(s)'); ylabel('Current␣(A)');
legend('I_{1a}','I_{1b}','I_{1c}','Orientation','
   horizontal');
grid on;axis fill;set(gca,'Fontsize',15);
saveas(gcf,'3
   f_droop_v1_corrientes_primera_viguales_desfase'
   ,'epsc');
figure();
plot((0:steps-1)*ts,U(1,:),'r-',(0:steps-1)*ts,U
   (2,:),'b-',(0:steps-1)*ts,U(3,:),'g-','
   LineWidth',2);
%title('Inputs');
xlabel('Time␣(s)'); ylabel('Voltage␣(V)');
legend('V_{1a}','V_{1b}','V_{1c}','Orientation','
   horizontal');
grid on;axis fill;set(gca,'Fontsize',15);
saveas(gcf,'3
   f_droop_v1_voltajes_primera_viguales_desfase','
   epsc');
figure();
plot((0:steps-1)*ts,Ps(1,:),'k-',(0:steps-1)*ts,Ps
   (2,:),'m-','LineWidth',2);
%title('Powers');
xlabel('Time␣(s)'); ylabel('Active␣Power␣(W)');
legend('P_{1_{3\phi}}','P_{2_{3\phi}}','
   Orientation','horizontal');
grid on;axis fill;set(gca,'Fontsize',15);
```

```
saveas(gcf,'3f_droop_Ppotencias_primera_viguales_desfase','epsc');
figure();
plot((0:steps-1)*ts,Qs(1,:),'b-',(0:steps-1)*ts,Qs(2,:),'r-','LineWidth',2);
%title('Powers');
xlabel('Time␣(s)'); ylabel('Reactive␣Power␣(VAR)');
legend('Q_{1_{3\phi}}','Q_{2_{3\phi}}','Orientation','horizontal');
grid on;axis fill;set(gca,'Fontsize',15);
saveas(gcf,'3f_droop_Qpotencias_primera_viguales_desfase','epsc');
figure();
plot((0:steps-1)*ts,W(1,:),(0:steps-1)*ts,W(2,:),'LineWidth',2);
%title('Frequencys');
xlabel('Time␣(s)'); ylabel('Frequency␣(rad/s)');
legend('\omega_1','\omega_2','Orientation','horizontal');
grid on;axis fill;set(gca,'Fontsize',15);
saveas(gcf,'3f_droop_Freq_primera_viguales_desfase','epsc');
end
if opt == 2
%%%%%%%--INDIVIDUAL PLOT::V2--%%%%%%%%
figure();
plot((0:steps-1)*ts,Salidas(4,:),(0:steps-1)*ts,Salidas(5,:),(0:steps-1)*ts,Salidas(6,:),'LineWidth',2);
%title('Outputs');
xlabel('Time␣(s)'); ylabel('Current␣(A)');
legend('I_{2a}','I_{2b}','I_{2c}','Orientation','horizontal');
grid on;axis fill;set(gca,'Fontsize',15);
saveas(gcf,'3f_droop_v2_corrientes_primera_viguales_desfase','epsc');
figure();
plot((0:steps-1)*ts,U(4,:),'r-',(0:steps-1)*ts,U(5,:),'b-',(0:steps-1)*ts,U(6,:),'g-','LineWidth',2);
```

```
%title('Inputs');
xlabel('Time (s)'); ylabel('Voltage (V)');
legend('V_{2a}','V_{2b}','V_{2c}','Orientation','
    horizontal');
grid on;axis fill;set(gca,'Fontsize',15);
saveas(gcf,'3
    f_droop_v2_voltajes_primera_viguales_desfase','
    epsc');
figure();
plot((0:steps-1)*ts,Ps(1,:),'k-',(0:steps-1)*ts,Ps
    (2,:),'m-','LineWidth',2);
%title('Powers');
xlabel('Time (s)'); ylabel('Active Power (W)');
legend('P_{1_{3\phi}}','P_{2_{3\phi}}','
    Orientation','horizontal');
grid on;axis fill;set(gca,'Fontsize',15);
saveas(gcf,'3
    f_droop_Ppotencias_primera_viguales_desfase','
    epsc');
figure();
plot((0:steps-1)*ts,Qs(1,:),'b-',(0:steps-1)*ts,Qs
    (2,:),'r-','LineWidth',2);
%title('Powers');
xlabel('Time (s)'); ylabel('Reactive Power (VAR)')
    ;
legend('Q_{1_{3\phi}}','Q_{2_{3\phi}}','
    Orientation','horizontal');
grid on;axis fill;set(gca,'Fontsize',15);
saveas(gcf,'3
    f_droop_Qpotencias_primera_viguales_desfase','
    epsc');
figure();
plot((0:steps-1)*ts,W(1,:),(0:steps-1)*ts,W(2,:),'
    LineWidth',2);
%title('Frequencys');
xlabel('Time (s)'); ylabel('Frequency (rad/s)');
legend('\omega_1','\omega_2','Orientation','
    horizontal');
grid on;axis fill;set(gca,'Fontsize',15);
saveas(gcf,'3f_droop_Freq_primera_viguales_desfase
    ','epsc');
end
toc
```